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**TWO-DIMENSIONAL NUMERICAL MODELLING OF INFLOWS TO A
DRIFT THROUGH MULTILAYERED PERMEABILITY GROUND**

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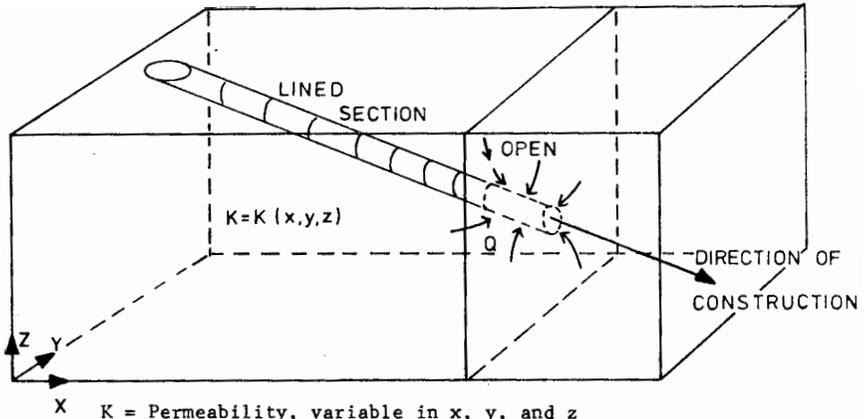
ABSTRACT

Digital computer finite-difference models have been developed to investigate steady-state inflows to a drift under construction. While it is accepted that totally adequate representation of drift inflows may only be achieved through use of three-dimensional flow models, preliminary work described in this paper has employed two-dimensional and radial models to approximate the problem. Both approximations were used to analyse flows towards a drift at different levels of construction in a sequence representative of the multilayered character of the coal-mining environment. The solutions obtained from each approximation demonstrated the essential differences in their formulation, although similar trends were identifiable. Inflow magnitude was found to be largely controlled by the permeability values of the layers in the sequence actually in contact with the open section of the drift. These permeability values were also found to strongly influence the distribution of any inflow over the sides of the open section. Provisional assessment of the validity of each approximation indicated that, of the two, the radial approximation was more likely to provide realistic inflow predictions.

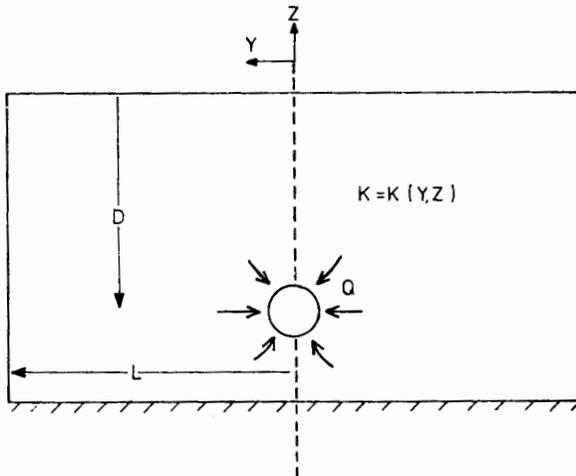
INTRODUCTION

Groundwater inflows to drifts under construction can lead to severe problems, both in practical and economic terms. Preventative action, such as grouting or freezing, may be undertaken to minimise these problems. However, such measures are costly, and the ability to predict accurately the depth zones in which they are needed is highly desirable. This requires an inflow assessment to be carried out which relates successive levels of drift construction to the expected water make at that depth.

Such an inflow assessment may, in principle, be carried out using mathematical modelling methods, providing that sufficient and reliable hydrogeological and geological data exist for the site in question. However, practical difficulties exist in representing drift inflows



X $K =$ Permeability, variable in x , y , and z
 $Q =$ Inflow to drift
 Fig. 1. Three-dimensional drift inflow problem



$D =$ Depth to centre of drift
 $L =$ Distance to outer boundary of model
 Fig. 2. Two-dimensional approximation for drift inflows

since there is no simple approximate geometric relationship between the inflow pattern and the drift itself. This suggests that the only totally adequate model in which drift inflows can be assessed involves solution for three-dimensional flow.

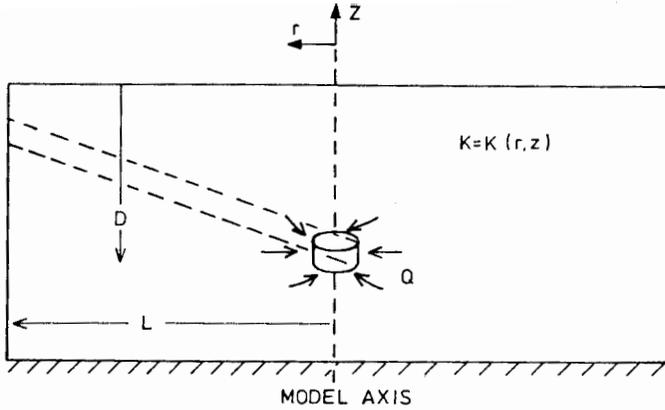
Since three-dimensional modelling is in many ways complex, preliminary work on drift inflows, described in this paper, attempts to approximate the problem through use of two-dimensional steady-state and radial steady-state digital models. Both models are used to investigate groundwater flows towards a drift situated at various levels in a multilayered permeability sequence, typical of ground found in the coal mining environment. Due to the approximate nature of the methods used, accurate inflow predictions are unlikely. However, the results obtained provide useful qualitative and semi-quantitative information on the characteristics of flows towards drifts in multilayered ground. Furthermore, the investigation enables some idea to be obtained of the extent to which the two models can approximate three-dimensional drift inflows.

MODELLING DRIFT INFLOWS

Inflows to underground excavations may be assessed using mathematical models, based on the relevant partial differential equation of flow derived from Darcy's Law and the principle of continuity. The assessment of drift inflows, however, poses a more complex problem than, for instance, the assessment of flows towards a shaft. This is due to the particular geometry of the situation which suggests that only a three-dimensional flow model may be totally adequate. Fig. 1 illustrates the general problem for flow through heterogeneous ground, and also indicates the method of construction that is assumed for modelling purposes throughout this study. This is taken as being one of progressive excavation and lining so that at any one time, the drift has a lined and open section. Flows are only considered towards the open section which may be of a length of up to 20 m. It is assumed that any inflow to the drift is quickly removed so that the pressure condition on the open section remains approximately atmospheric or zero. This feature is useful in the formulation of the problem, described later.

It is also important to discuss the time-dependence of drift inflows. It is arguable (Parsons, 1983) that since construction rates are relatively slow, then pressure changes in the surrounding aquifer must also be slow. Hence non-steady-state inflows may not be much greater than steady-state flows. This will depend on factors such as permeability layering, confining situations, and the storage properties of the ground, and situations can be envisaged where the above hypothesis may not be correct. Although this topic should form part of future work, the investigations described in this paper have been carried out using steady-state models.

The problem illustrated in Fig. 1 is complex and while a three-dimensional model should eventually be developed, preliminary investigations on the behaviour of flows towards drifts in multilayered permeability ground can be carried out using two-dimensional and radial approximations. Both these approaches are discussed below.



D = Depth to centre of opening representing open section
 L = Radial distance to outer boundary of model
 Fig. 3. Radial approximation for drift inflows

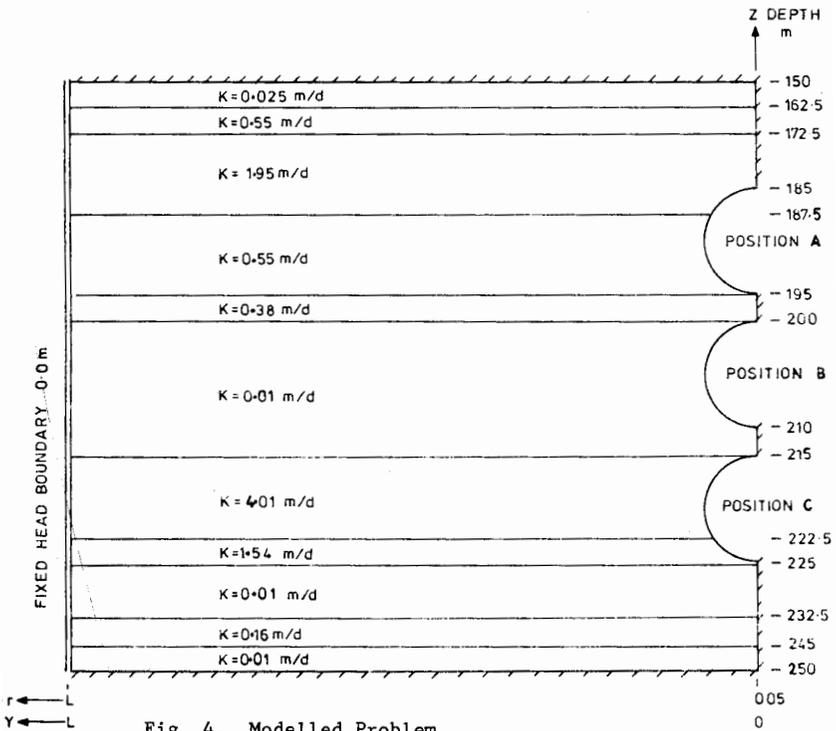


Fig. 4. Modelled Problem

Two-Dimensional Approximation

This involves the assessment of flows in two-dimensions ($y-z$) in a vertical section of unit width perpendicular to the axis of the drift (Fig. 2), the downward gradient of which is not considered. While this form of approximation may be good for a very long structure, such as an unlined tunnel, it may be inaccurate when applied to the open section of a drift where lengths are unlikely to be greater than 20 m. No consideration is made of the inflows to the ends of the open section, and the likely convergent nature of the flows in y , z , and x , is not represented.

Radial Approximation

In this method, the open section of the drift is approximated as a suitably dimensioned ellipsoidal or cylindrical opening while the lined section is ignored (Fig. 3). Flow towards this opening can then be analysed using radial flow ($r-z$) models. This form of approximation has the advantage that flow is represented in three-dimensions, if only axisymmetrically, and so should give more realistic inflow values. However, the geometry of the open section is poorly represented since inflows occur at an equal rate at any height all around the opening.

The following section describes the drift inflow problem to which these two approximations have been applied.

MODELLED PROBLEM

The modelled problem is illustrated in Fig. 4 and consists of a 100 m thickness of horizontally layered ground of varying permeability where the top of the region lies at a depth of 150 m below datum. The permeability data are based on those collected from the Warwickshire Coalfield in the U.K., and have been published, together with a subsequent inflow analysis for a proposed shaft and drift, in Lloyd *et al.* (1983). The lateral boundaries of the modelled region are set at a distance L from the centre of the drift.

For the two-dimensional approximation this implies symmetry about a vertical line through the centre of the drift across which no flow passes, and for the radial approximation the modelled region is cylindrical with radius L . The permeability layering extends horizontally throughout both modelled regions and within each layer, the permeability is equal in the vertical and horizontal or radial directions. Flows are considered to drifts at three levels in turn, at positions A, B, and C. These correspond to different stages in construction. In the two dimensional approximation the modelled drift approximates to a circular cross-section of diameter 10 m, while in the radial approximation the open section approximates to an ellipsoid with a 10 m vertical height and a 5 m radius to the largest circumference.

TWO-DIMENSIONAL FORMULATION OF PROBLEM

This includes a statement of the equation of flow together with the necessary boundary conditions to solve the problem. Steady flows in the $y-z$ plane are described by the following flow equation:

$$\frac{\partial}{\partial y} (K(y) \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (K(z) \frac{\partial h}{\partial z}) = 0$$

where K = Permeability of ground, variable in y and z
h = Groundwater Head potential

Boundary conditions for the y-z section are shown schematically in Fig. 4 and are as follows. Flows towards the drift are assumed to be derived laterally and hence a fixed groundwater head boundary is set at a distance L either side of the drift. The fixed head on these boundaries is set at 0.0 m (i.e. under hydrostatic pressure). This produces a vertical line of symmetry through the centre of the drift across which no flow passes. Hence only half the complete section need be modelled and is shown in Fig. 4. The internal boundary condition on the drift consists of a fixed head distribution where the head is equal to the depth below datum. This results from the pressure condition within the shaft being assumed as atmospheric as discussed previously. The top and bottom boundaries to the section are of the no-flow type and may correspond to extremely low permeability bands just above and below the section.

RADIAL FORMULATION OF PROBLEM

The governing equation in this case is:

$$\frac{1}{r} \frac{\partial}{\partial r} (r K(r) \frac{\partial h}{\partial r}) + \frac{\partial}{\partial z} (K(z) \frac{\partial h}{\partial z}) = 0$$

where K = Permeability, variable in r and z

Boundary conditions for this approximation are similar to those for the y-z approximation with a fixed head outer boundary set at 0.0 m at a radial distance L. The inside radial boundary is set at a radius of 0.05 m due to the logarithmic nature of the model and is of the no-flow type except on the drift opening where the fixed head condition described above applies.

DIGITAL COMPUTER MODELS

Solutions to the problems outlined above can be obtained using digital computer models. The numerical technique employed in both the two-dimensional and radial models is the standard finite-difference approximation for which the underlying theory and advantages are given in Rushton and Redshaw (1979). Essentially, the technique involves discretising the modelled region using a finite-difference mesh where each mesh node represents a given volume. The finite-difference form of the equation of flow is then solved iteratively for each mesh node until the residual error at each node is less than a pre-chosen value. The iterative method of solution used in both these models is the Strongly Implicit Procedure (SIP) which has been found to reach solutions efficiently in terms of computer time. Early trials using the simpler iterative procedure of Successive Over Relaxation (SOR) were found to be computationally inefficient, especially if the modelled ground conditions were complex or the flow radial in type. SIP was first developed by Stone (1968) and has been evaluated against other iterative methods by Trescott and Larson (1977). Attempts were made to make the finite-difference grid as representative as possible of the modelled region and

the likely flow regime. Hence irregular grids were used with higher nodal concentrations in the region of the drifts. In the radial model, a logarithmic transformation was applied to the governing equation which produces equal logarithmic spacing in the radial direction and hence greater nodal concentration close to the drift where r is small. All models were run so that the maximum residual error in the finite difference form of the governing equation was 10^{-6} .

DRIFT INFLOW RESULTS USING THE TWO-DIMENSIONAL APPROXIMATION

The drift inflow results obtained using the two-dimensional approximation are shown in Table 1 for different values of L , the distance to the outer fixed head boundary. The units of flow are m^3/d per m open section, so that on the basis of this approximation an open section of 10 m length would experience an inflow ten times that shown.

L(m)	INFLOWS (m^3/d per m OPEN SECTION)		
	POSITION A	POSITION B	POSITION C
2000	13**	13*	
1000	25**	24*	
464	46*	40	See
316	62*	49	Text
100	158	83	

* Free surface present
 ** Free surface intersects drift

Table 1 Inflow values for different values of L
 (Two-dimensional Approximation)

Steady-state solutions obtained for large L show the formation of unsaturated zones towards the top of the modelled section. Free surfaces were located using a trial free boundary method and the general form of the solutions is shown in Fig. 5. The extent of the unsaturated zone was found to increase with increasing L , and also tended to be larger where the permeabilities of the layers enclosing the drift were relatively high. Free surfaces intersect the drift for very large L , and as L increases this point of intersection tends towards the controlling head at the base of the drift.

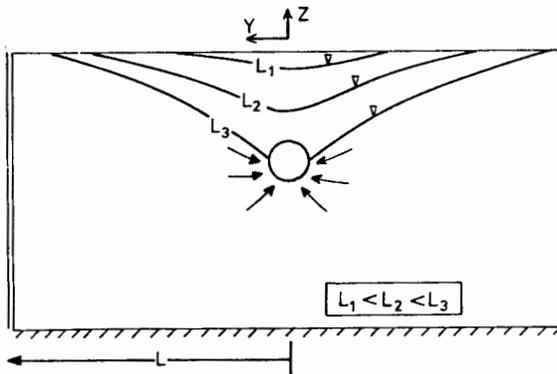


Fig. 5. General form of solution (Two-dimensional Approximation), showing influence of L on free surface

This phenomenon creates difficulties in the interpretation of the results. For instance, inflows to the drift at position A (Fig. 4) would be expected to be greater than those to B due to the relative permeability values of the ground surrounding the drifts. However, for $L = 1000$ m, inflows to A and B are almost equal. In the case of A, this is due to the exclusion of any inflow from the layer of permeability 1.95 m/d (Fig. 4), since the free surface intersects the drift beneath this layer. The free surface in case B lies some way above the drift due to the low permeabilities of the drift-enclosing layers, and hence inflows can occur through the relatively high permeability layer of 0.38 m/d. However, for small L and saturated conditions throughout the modelled region, inflows to A are almost twice those to B.

Inflows to the drift at position C were hard to analyse due to the presence of unsaturated areas in the vicinity of the drift (Fig. 6). This is probably due to high inflows not being compensated for by flow through the low permeability layer above the drift and is a similar effect to that described above.

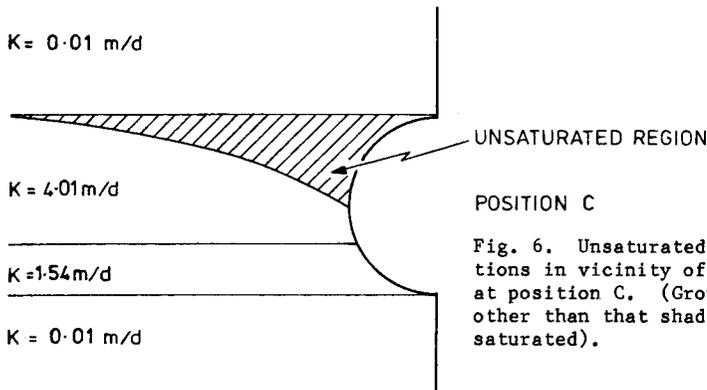


Fig. 6. Unsaturated conditions in vicinity of drift at position C. (Ground other than that shaded is saturated).

The influence of L on inflows is considerable, the smallest percentage rise in inflow over the range $1000 \text{ m} > L > 100 \text{ m}$ being 246 percent (Position B). As L becomes large, inflows tend to zero, and as L decreases inflows increase rapidly. This sensitivity to model boundary conditions is a major disadvantage of the two-dimensional approximation for assessing inflows to drifts.

DRIPT INFLOW RESULTS USING THE RADIAL APPROXIMATION

The drift inflow results obtained using the radial approximation are shown in Table 2. The modelled region remained saturated throughout

L(m)	INFLOWS (m^3/d)		
	POSITION A	POSITION B	POSITION C
5000	7160	1690	9330
1000	8190	1740	10860
500	8740	1760	11700
232	9580	1790	13070
107	10970	1830	15640

Table 2 Inflow values for different values of L (Radial Approximation)

in all cases considered due to the convergent nature of flow inherent in the radial model. Inflows are far higher than those obtained from the two-dimensional approximation. As would be expected, results show good correlation between inflow magnitude and the relative permeabilities of the drift enclosing layers. Inflows to B would be expected to be still lower were the top of the drift not in contact with the layer of permeability 0.38 m/d through which up to 85 percent of the inflow is derived. Similar variations in inflow distribution over the sides of the drift also occur in cases A and C (Fig. 7).

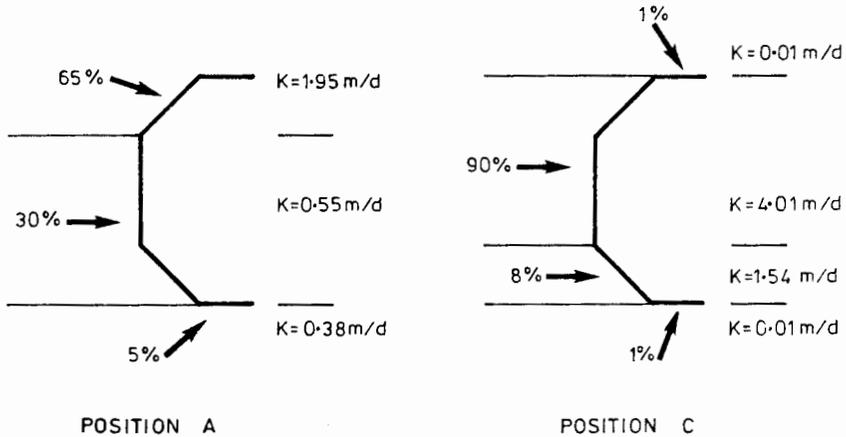


Fig. 7. General distribution of inflow over sides of drift at positions A and C. (Radial Approximation)

The influence of L on the inflows is not as great as is found using the two-dimensional approximation, although this varies with the permeabilities of the drift-enclosing layers. Hence the largest variation occurs in inflows to C (44 percent increase over the range $1000\text{ m} > L > 100\text{ m}$), while inflows to B show only slight variation. Thus, the radial approximation allows inflow predictions within a fairly reasonable range, and is less sensitive to boundary effects than the two-dimensional approximation.

DISCUSSION

Quite different results have been obtained from use of each approximation in terms of inflow magnitude, the influence of the outer model boundary, and the formation of unsaturated zones within the modelled region (e.g. Fig. 8). For instance, even for a 20 m open section, inflows obtained from the two-dimensional approximation are an order of magnitude lower than those obtained from the radial approximation. This demonstrates the different geometries of the models used. While the two-dimensional approximation considers two-dimensional flow in a vertical section towards a circular opening, the radial approximation analyses axisymmetric conver-

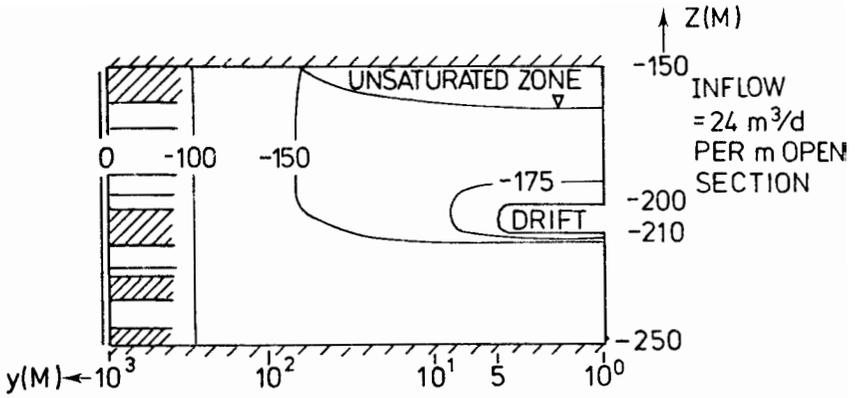


Fig. 8(a). Two-dimensional Approximation

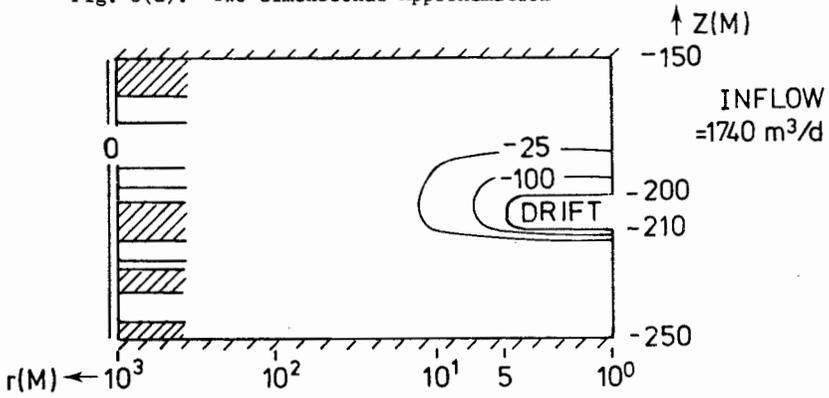


Fig. 8(b). Radial Approximation

Fig. 8. Groundwater head distribution (m) for inflows to drift at position B ($L = 1000$ m).

gent flow to an ellipsoidal opening. The respective groundwater head distributions in the modelled region illustrate this difference, and are shown in Fig. 8 for the case $L = 1000$ m and the drift at position B. Head loss across the modelled region typically occurs nearer, and around, the drift in the radial approximation and this is a characteristic of the governing radial flow equation. The far more gradual head loss experienced in two-dimensional analysis leads to greater sensitivity to the outer fixed-head boundary and the formation of unsaturated zones towards the top of the modelled section.

Flows to the open section of a drift are expected to be three-dimensionally convergent, though not necessarily axisymmetric about some vertical axis through the drift. Two two-dimensional approximation does not consider horizontally convergent flow, especially that to both ends of the open section of the drift. Corrections for these inflows have been

applied and reasonable solutions obtained (Lloyd *et al.*, 1983). It is therefore accepted that the analysis of two-dimensional flows solely perpendicular to a drift may not give realistic inflow values. However, other uses for this method may exist, including the estimation of inflows to the complete length of an unlined drift, or for the qualitative study of the effect on inflows of subsurface features such as fissures.

The radial approximation is felt to give more realistic values for inflows to the open section of a drift under construction. An idea of the order of inflow magnitude expected in the multilayered ground investigated in this study can be obtained from the shaft inflow data presented in Lloyd *et al.* (1983). This was recorded during shaft construction in ground adjacent to the site where the permeability data used in this study was obtained and the inflow values vary between 1000 and 18000 m³/d. The drift inflows obtained in this study from the radial approximation fall within this range. However, while this method has produced promising results, more work is needed to fully assess its validity, especially in terms of the geometric representation of the open section.

CONCLUSIONS

The analysis of flows towards a drift under construction is essentially a three-dimensional problem. However, the two approximations used in this study provide a useful basis for further research. For the case of drift construction through multilayered permeability ground, results have been obtained that indicate the variation in inflow that may occur with depth. Inflow magnitude is largely controlled by the permeability values of the particular layers in contact with the open section of the drift, and these values also strongly influence the distribution of inflow over the sides of the working.

Two-dimensional analysis of flows, solely perpendicular to the drift, is unlikely to give realistic inflow values, although other applications of this method may provide useful qualitative information. Approximating the flows towards a drift as being radial axisymmetric about a vertical axis through the open section is thought to provide more realistic inflow values, and future work should aim to fully assess the validity of this approximation. However, the geometry of the drift open section is poorly represented in the radial approximation and adequate analysis of flows near to the drift may require the development of a three-dimensional model.

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