Water Hammer Analysis of Pumping Systems for Control of Water in Underground Mines

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ABSTRACT

This paper considers water hammer analysis of pumping systems for control of water in underground mines. The basic mechanisms causing water hammer events in pipe systems are introduced. Expressions for the wave speed in both an infinite fluid and in a thin-walled pipeline is presented. The equations of unsteady flow in pipelines and the method of characteristics solution to these equations are described. Methods for controlling water hammer in pipelines are described. Two boundary conditions are discussed including the reservoir and the pump. A case study for a pumping system in an underground mine in Velenje, Yugoslavia is presented in detail. Field measurements are compared with a computer simulation analysis of a transient during power failure to the pump. The results show that the method of characteristics is an acceptable method for water hammer analysis of mine pumping systems.

INTRODUCTION

The control of water hammer pressures in pumping systems is essential for economic and safe operation of underground mines. Water hammer may be caused by a number of different events including start-up, power failure to pump motors, pump run down and opening and closing of valves in the pipeline. Pumps in mines are usually centrifugal pumps (1). It is preferable to pump nearly-clean water (very low-concentration ratio of solids) to minimise pump wear. Suspended solids should be removed prior to pumping water if possible.

This paper deals with water hammer analysis of a pumping system delivering nearly-clean water to the surface for the purpose of dewatering a mine. In the first part of the paper a review of the basic solutions of the hyperbolic partial differential equations governing water hammer events in pipelines is presented. Two boundary conditions are described. Several design approaches to control water hammer in pipelines are depicted. Details of a case study of the analysis of a pumping system in a mine at Velenje, Yugoslavia are then discussed. During the design analysis special attention should be given to water hammer events in the system.
A water hammer event or hydraulic transient results when the velocity of flow changes in a pipeline. Water hammer is the transmission of pressure waves along pipelines resulting from a change in flow velocity. When the steady flow of an elastic fluid in a pipe is disturbed (for example, opening or closing a valve in a pipeline) the effect is not felt immediately at other points in the pipeline. The effect is transmitted along the pipe at a finite velocity called the wave speed of the fluid.

Typical causes of water hammer include the adjustment of a valve in a piping system, starting or stopping of a pump, and load rejection of a turbine in a hydro-electric power plant. Water hammer in systems is becoming increasingly important as technology advances, larger equipment is constructed, and higher speeds are employed for pumps and turbines. Possible outcomes of water hammer events include dangerously high pressures, excessive noise, fatigue, pitting due to cavitation, disruption of normal control of circuits, and the destructive resonant vibrations associated with the inherent period of certain systems of pipes.

The objective of water hammer analysis is to calculate the pressures and velocities during an unsteady-state mode of operation. The analysis of unsteady flow is much more complex than for steady flow. Another independent variable, that of time, enters and the resulting equations are partial differential equations rather than ordinary differential equations. The solution of the resulting hyperbolic partial differential equations by the method of characteristics is well suited to the speed and accuracy of digital computers.

**WAVE SPEED THROUGH A FLUID**

The wave speed \( a \) in an infinite fluid is given as:

\[
a = \sqrt{\frac{K}{\rho}}
\]

where \( \rho \) is the density of the fluid and \( K \) is the bulk modulus of elasticity of the fluid. For water at 20 degrees Celsius \( a = 1485 \) m/s for \( \rho = 998.2 \) kg/m\(^3\) and \( K = 2.2 \times 10^9 \) N/m\(^2\). This would be the maximum wave speed that would be expected to occur in a pipeline filled with water.

For fluid in a pipeline the elasticity of the pipe walls reduces the wave velocity. From the unsteady continuity equation \(^{(4)}\) for a pipeline it can be shown that:

\[
a^2 = \frac{K/\rho}{1 + (K/A)(\Delta A/\Delta p)}
\]

where \( \Delta A \) is the change in area of the pipe corresponding to a change in pressure \( \Delta p \), and \( A \) is the cross-sectional area of the pipe. Thus wave speed is a function of elasticity of the pipeline as reflected by the \( \Delta A/\Delta p \) ratio or the area change of the pipe for a given pressure change.

The hoop stress and strain relations can be introduced to obtain expressions for the wave speed in thin walled pipelines. Consider a pipeline of diameter \( D \) and wall thickness
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δ with a steady state pressure of $p_0$. A thin walled pipeline is defined as one for which the following holds:

$$\frac{D}{\delta} > 25$$

(3)

A general expression for the wave speed in a thin walled pipeline is:

$$a = \sqrt{\frac{K/\rho}{1 + [(K/E)(D/\delta)]c_1}}$$

(4)

where $E$ is Young’s modulus of elasticity of the pipe wall material, and $c_1$ depends on the pipe restraint as follows:

- Case a. For a pipe anchored at the upstream end only

$$c_1 = \left(1 - \frac{\nu}{2}\right)$$

(5)

where $\nu$ is Poisson’s ratio for the pipe wall material.

- Case b. For a pipe anchored throughout its length

$$c_1 = (1 - \nu^2)$$

(6)

- Case c. For a pipe with expansion joints throughout its length

$$c_1 = 1$$

(7)

EQUATIONS OF UNSTEADY FLOW

The simplified equation of motion for unsteady flow\(^{(4)}\) for a pipeline is:

$$\frac{1}{A} \frac{\partial Q}{\partial t} + g \frac{\partial H}{\partial x} + \frac{f}{2DA^2} Q|Q| = 0$$

(8)

The simplified continuity equation for unsteady pipeline flow is:

$$\frac{\partial H}{\partial t} + a^2 \frac{\partial Q}{gA \partial x} = 0$$

(9)

in which the dependent variables are the piezometric head or hydraulic grade line elevation $H$ with respect to a specified horizontal datum, and the discharge $Q$ at a section. In addition $g$ is the gravitational acceleration, $f$ is the Darcy–Weisbach friction factor, and $x$ and $t$ are the independent variables denoting distance along the pipeline and time.

Eqs. 8 and 9 are a set of quasi-linear hyperbolic partial differential equations. There are 2 dependent variables that are required to be solved for in order to obtain a solution to the transient problem. These are the hydraulic grade line elevation or head $H(x, t)$ and the discharge $Q(x, t)$. A general solution to these partial differential equations is not available.
THE METHOD OF CHARACTERISTICS TRANSFORMATION

The common method of solving Eqs. 8 and 9 is by the method of characteristics transformation. The two partial differential equations are transformed to four ordinary differential equations. The two ordinary differential compatibility equations are:

$$\pm g \frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0$$

(10)

Each compatibility equation is only valid along its corresponding characteristic line (Figure 1) given as:

$$\frac{dx}{dt} = \pm a$$

(11)

Eqs. 10 may be integrated along their respective $C^+$ and $C^-$ characteristic lines in Figure 1 to provide the standard water hammer compatibility equations. The method of specified time intervals is used. The integrated compatibility equation for the $C^+$ line is:

$$H_P - H_{i-1} + \frac{a}{gA} (Q_P - Q_{i-1}) + \frac{f\Delta x}{2gDA^2} |Q_{i-1}|Q_P = 0$$

(12)
where \( Q_{i-1} \) is the known discharge at the immediately adjacent upstream section at time \( t - \Delta t \), \( H_{i-1} \) is the known hydraulic grade line elevation at the immediately adjacent upstream section at time \( t - \Delta t \) (Figure 1). \( H_P \) and \( Q_P \) are the unknown hydraulic grade line elevation and discharge for the current time \( t \). Eq. 12 is only valid along the \( C^+ \) characteristic line given from Eq. 11 as:

\[
\frac{\Delta x}{\Delta t} = a
\]  

where \( \Delta x \) is the reach length and \( \Delta t \) is the time step (Figure 1). Eq. 13 is referred to as the Courant condition. This provides a fixed relationship between the pipeline discretization selected and the time step used for computations using the method of characteristics.

For the \( C^- \) characteristic line the integrated compatibility equation is:

\[
H_P - H_{i+1} - \frac{a}{gA} (Q_P - Q_{i+1}) - \frac{f \Delta x}{2gDA^2} |Q_{i+1}| Q_P = 0
\]  

where \( Q_{i+1} \) is the known discharge at the immediately adjacent downstream section at time \( t - \Delta t \) and \( H_{i+1} \) the known hydraulic grade line elevation at the immediately adjacent downstream section at time \( t - \Delta t \). The friction term in each of the compatibility equations has been obtained by using an integration by parts method described by Wylie(5). Eq. 14 is only valid along the \( C^- \) characteristic line. Column separation is taken into account if the HGL is computed to be below the vapour head at a section.

**BOUNDARY CONDITIONS**

**Introduction**

There are many boundary conditions for which the equations have been previously developed\(^{(4,6)}\). These include reservoirs, dead ends, valves, pumps, pipe series connections, pipe branch connections, turbines, air chambers, surge tanks, pressure relief valves, and discrete vapour and gas cavities\(^{(7)}\). Two of the more common boundary conditions will be now considered.

**The Upstream Reservoir Boundary Condition**

The conditions at an upstream reservoir are influenced by the conditions at the section immediately downstream. Thus the \( C^- \) integrated compatibility equation (Eq. 14) is used which is valid along the \( C^- \) characteristic line. This equation brings information to the reservoir from the computational section adjacent to reservoir at the previous time step. There are 2 unknowns in this equation including the head \( H_P \) and discharge \( Q_P \) at the reservoir. In Eq. 14, all the other variables depend on the known conditions at the section in the pipeline adjacent to the reservoir at the previous time step as seen previously. There is no positive \( C^+ \) characteristic for the reservoir (from the left hand side). One of the unknown variables for the reservoir is however always specified. The reservoir head \( H_P \) is constant

\[
H_P = H_R
\]  

The unknown discharge \( Q_P \) at the reservoir is then calculated from Eq. 14 using \( H_P \) from Eq. 15. Thus to alter the discharge at the reservoir then either the head \( H_{i+1} \) or the discharge \( Q_{i+1} \) must altered by changing conditions at the downstream end of the pipeline.
The Pump Boundary Condition

The pump boundary condition is much more complicated than the reservoir boundary condition. There are 5 unknowns including

- $Q$, the pump discharge
- $H_{P_s}$, HGL on suction side of pump
- $H_{P_d}$, HGL on discharge side of pump
- $N$, rotational speed of pump
- $T$, pump torque

Five non-linear simultaneous equations result based on the $C^+$ and $C^-$ equations upstream and downstream of the pump, the moment of momentum or torque equation for the pump, the head versus discharge curve, and the torque versus discharge curve. These last two curves are usually given in dimensionless form. The measured pump rotational speed curve versus time can be used instead of the torque versus discharge curve when analysing results of measurements and calculations in a pump system. Space does not permit the full presentation of details of the solution of the pump boundary condition. Streeter and Wylie\textsuperscript{(4)} gives full details. The 5 non-linear equations describing the pump boundary condition may be solved using Newton’s method.

CONTROL OF WATER HAMMER IN PIPELINE SYSTEMS

Water hammer caused by the start-up, or stoppage of pumps, pump run down, and opening and closing of valves in the pipeline is manifested as high pressure fluctuations and possible column separation in the system. Other possible effects are excessive reverse pump rotation and check valve slam. The undesirable water hammer effects may disturb overall operation of the system and damage components of the system, for example pipe rupture. Therefore several design approaches may be adopted to solve water hammer problems:

- Installation of surge control devices in the system. Table 1 shows a summary of various water hammer control devices which may be installed in the system\textsuperscript{(9)}.
- Redesign of the pipeline layout e.g. change of elevation, length or diameter of the pipeline.
- Design of a thicker pipeline or selection of a pipe material with higher strength to allow column separation in the system.
- Alteration of operational parameters e.g. reduction of velocity in the pipeline.

Economic and safety factors are decisive for the type of protection against undesirable water hammer effects. A number of alternatives should be considered before final design which may include a combination of various design approaches.
### Table 1. Summary of waterhammer control devices

<table>
<thead>
<tr>
<th>Device</th>
<th>Schematic</th>
<th>Principle of operation</th>
<th>Pipeline system/efficiency</th>
<th>Protection against</th>
<th>Reliability</th>
<th>Additional inertia</th>
<th>Frequency of application</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rupture disk</td>
<td><img src="image" alt="Image" /></td>
<td>Relieves pressure</td>
<td>High head systems</td>
<td>High pressures, column separation</td>
<td>Excellent</td>
<td>None</td>
<td>Rarely</td>
<td>Very high</td>
</tr>
<tr>
<td>Air valve</td>
<td><img src="image" alt="Image" /></td>
<td>Maintains flow, controls admission and release</td>
<td>Long pipe with high points</td>
<td>High pressures, column separation and column separation</td>
<td>Good</td>
<td>None</td>
<td>Sometimes</td>
<td>Low</td>
</tr>
<tr>
<td>Bypass</td>
<td><img src="image" alt="Image" /></td>
<td>Relieves pressure</td>
<td>High head systems</td>
<td>High pressures, column separation</td>
<td>Excellent</td>
<td>None</td>
<td>Sometimes</td>
<td>Low</td>
</tr>
<tr>
<td>One-way surge tank</td>
<td><img src="image" alt="Image" /></td>
<td>Provides flow</td>
<td>Long pipe, medium to high head systems</td>
<td>Long pipe, medium to high head systems</td>
<td>Excellent</td>
<td>None</td>
<td>Check-in-line valve</td>
<td>Occasionally</td>
</tr>
<tr>
<td>Air chamber surge tank</td>
<td><img src="image" alt="Image" /></td>
<td>Energy accumulator</td>
<td>Long pipe, medium to high head systems</td>
<td>Long pipe, medium to high head systems</td>
<td>Excellent</td>
<td>None</td>
<td>Check-in-line valve</td>
<td>Occasionally</td>
</tr>
<tr>
<td>Surge tank</td>
<td><img src="image" alt="Image" /></td>
<td>Energy accumulator</td>
<td>Very low head systems</td>
<td>Very low head systems</td>
<td>Always useful</td>
<td>None</td>
<td>Check-in-line valve</td>
<td>Occasionally</td>
</tr>
<tr>
<td>Controled valve closure</td>
<td><img src="image" alt="Image" /></td>
<td>Regulates discharge</td>
<td>Always</td>
<td>Always</td>
<td>Always</td>
<td>None</td>
<td>Check-in-line valve</td>
<td>Occasionally</td>
</tr>
<tr>
<td>Additional inertia</td>
<td><img src="image" alt="Image" /></td>
<td>Lengthens rundown time</td>
<td>&lt;2000 m</td>
<td>&lt;2000 m</td>
<td>&lt;2000 m</td>
<td>None</td>
<td>Check-in-line valve</td>
<td>Occasionally</td>
</tr>
</tbody>
</table>

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A computerized mathematical model for the water hammer and column separation analysis is applied to an underground mine pumping system in Velenje, Yugoslavia. The pump system is a high head system with a horizontal multistage centrifugal pump equipped with a check valve forcing water into a nearly vertical pipeline discharging into an atmosphere (Figure 2).

The pump normally operates at the following conditions:

- pump head $H = 382$ m
- Discharge $Q = 0.05 \text{ m}^3/\text{s}$
- Rotational speed $n = 1485$ rpm

![Figure 2: Envelopes of maximum and minimum hydraulic grade line (HGL) along the pipe after pump power failure](image)

The pipeline data are as follows:

- length of the pipe $L = 441.5$ m
- internal pipe diameter $D = 0.205$ m
- wall thickness of steel pipe $\delta = 0.007$ m
The maximum concentration ratio of solids in water $\alpha = 0.01$. Because the concentration ratio $\alpha$ is very small the mathematical model for a one-phase water hammer with column separation described previously is applied.

A computer analysis by the method of characteristics was carried out for a pump start-up and run down for operating conditions which appear in-situ. To confirm acceptability of the model, measurements at various operating conditions were performed. The following variables were measured:

- pressure at suction side of the pump
- pressure at delivery side of the pump both at the upstream and the downstream end of the check valve
- rotational speed
- check valve closure time

![Figure 3: Dimensionless measured pump rotational speed after power loss](image)

All measured data were recorded on multi-channel recorder.

A detailed analysis of the results of computer model calculations and field measurements for a pump run down at pump head $H = 382$ m, discharge $Q = 0.05$ m$^3$/s and rotational speed $n = 1485$ rpm is presented. Input data included the measured wave speed $a = 1318$ m/s, the total check valve closure time $T_c = 1.1$ s, and values of pump rotational speed during pump run down $n/n_0$ - see Figure 3.

The measured wave speed $a = 1318$ m/s agrees very well with theoretical one which is calculated by the Eqs. 4 and 6:
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\[ a = \sqrt{\frac{K/\rho}{1 + \frac{K D}{E \delta (1 - \nu^2)}}} = \sqrt{\frac{2.19 \times 10^9 / 999}{1 + \frac{2.19 \times 10^9 \times 0.205}{2.06 \times 10^{11} \times 0.007 (1 - 0.27^2)}}} = 1304.3 \text{ m/s} \]

The total measured stoppage time of the pump after power loss was \( T_s = 40 \) s. The measured decrease of pump rotational speed used in computer model calculation represents pump behaviour during transients including inertial effects of the pump, clutch and electromotor. The envelopes of the calculated maximum and minimum hydraulic grade line (HGL) along the pipeline profile (EL) are shown on Figure 2. The diagram is important for pipeline designer to construct safe and economic system. As it may be seen from Figure 2 there is a distributed vaporous cavitation at the downstream end of the pipeline which does not significantly affect the shape of both envelopes.

Figure 4: Calculated and measured HGLs immediately downstream of the check valve

Figure 4 shows a comparison between the results of calculation and measurement for HGL immediately at the downstream side of the check valve connected to the pump. The calculated and measured maximum and minimum HGL are in good agreement which indicates that the method of characteristics is an acceptable method for water hammer and column separation analysis in pumping systems. However there are some discrepancies between the results of measurement and calculation which does not significantly affect the main design parameters i.e. the maximum and the minimum HGL. As it may be seen from the Figure 4 there is a slight time shift between the calculated and measured curves of HGL. The difference is mainly due to difficulties in the modelling of the hydrodynamic behaviour of the check valve influenced by internal and external forces\(^{(10)}\). The attenuation of the measured HGL is larger than that of
computer model calculation because in the calculation, a steady state turbulent friction term was used. An unsteady state turbulent friction term is still in stage of development\(^{(11,12,13)}\). However, experimental analysis and developed semi–empirical models show that the friction term in unsteady turbulent flow is larger than the one in steady turbulent flow\(^{(14)}\). On the contrary, the mathematical model for unsteady laminar friction term has been developed\(^{(15,16)}\). A slight effect of calculated distributed discrete water column separation at the upper part of the pipeline may be seen at the first and the second peaks from Figure 4, after that the disturbance is completely attenuated. No evidence of cavitating flow can be found from the results of measurements. That is why the effect of distributed continuous water column separation is much less than the effect of distributed discrete water column separation predicted by calculation\(^{(17,18,19)}\). In the case of severe water column separation, transient pressure pulses with jagged curve would be indicated\(^{(20)}\).

**CONCLUSIONS**

Results of computer model calculations and field measurements show that the method of characteristics is an acceptable method for water hammer analysis for control of water in the underground mine. Very good agreement is obtained for a maximum and a minimum pressure head, the two important parameters for pipeline design. However the analyst should be aware of discrepancies which may arise due to simplifications in numerical analysis. Thorough analysis of the example presented in the paper show that further work in a hydrodynamic modelling of a check valve influenced by internal and external forces, an estimation of friction term in unsteady turbulent flow, and simulation of transient cavitating flow, is needed.

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