Density - dependent calculation of matrix fracture flow

Torsten Seidel, Christoph König

delta h Ingenieurgesellschaft mbH, Parkweg 67, 58453 Witten, Germany

Abstract In solid rock aquifers low matrix permeability is often accompanied with fractures and faults of different scales. Therefore, numerical modelling of such coupled systems is very complex. Density dependent flow of groundwater and mass transport processes are coupled in fractured systems. These processes have to be modelled along discrete surfaces joined to the common matrix representation. This leads to high demands on the quality of finite element mesh generation. This paper presents a strategy to calculate three dimensional finite element meshes, which allows the modelling of density dependent groundwater flow and contaminant transport in a coupled matrix fracture aquifer system.

Key Words fracture flow, elder problem, variable-density flow and transport, stochastic fracture generation

Introduction

The goal of this paper is the investigation of the barrier property of the area close to the underground disposal sites in rock. The processes which occur in density-dependent groundwater flow and mass transport have to be tested numerically on a local model which covers a distance of a few hundred meters. The rock matrix with microfractures (for example shale and siltstone series) can be described as a porous continuum with anisotropic permeability (1/3), depending on the structure of the microfractures. It does not make sense to realize a deterministic approximation of the microfractures in a local model, nor is it manageable from a technical point of view. However, the discrete modelling of fractures and fissures is of primary importance for a realistic local model. The local effects on the distribution of pore pressure and water contents cannot be represented by a homogenization of the fractures to an additional porous continuum (double porosity). For example, it is not possible to approximate the tailing effects and the sharp density and concentration front using by double porosity models. Often three-dimensional fracture matrix systems are reduced to a two-dimensional model by using vertical cross-sections. This two-dimensional description of flow processes in fractured layers neglects the flow paths which arise from connections between intersected fractures in space. Therefore, the model will always underestimate the true conductivity of these layers. For that reason, a generator for stochastic three-dimensional fractures and a mesh generator that links these fractures to the rock matrix (SPRING, 2010) which is able to calculate the coupled system in three dimensions was developed.

Physical Processes and Equations in solid rock

Groundwater flow takes place in the fracture network where advection and gravity forces are the dominant process. For characterizing the flow and transport phenomena in a fractured rock on a local scale, it is necessary to model a discrete fracture matrix system. Concerning the flow, the fractures are assumed to be and parallel plates. The flow is considered to be laminar and the pressure to be constant across the aperture of fracture area.

This results in a neglection of the component of velocity, which is normal to the fracture plane. With this premises the cubic law with conductivity proportional to the square of the aperture b can be used:

$$v_k = -\frac{b^2}{12} k_{rel} \frac{\varsigma g}{\mu} \frac{\partial}{\partial x_k}\left(\frac{p}{\varsigma g} + z\right) \qquad k = r,s$$

The subscript k represents the local coordinates r, s which describe the fracture plane (Fig. 1). Gravitational force g acts into the global z-direction.

By intercalculating the flow equations to the mass balance, we obtain the flow equations for the porous rock matrix which included the generalized Darcy law. In the density dependent case $\varsigma = \varsigma(c)$ the flow equation is coupled with the transport equation for the solute concentration c. Fixing initial and boundary conditions for both mass balance equations will complete the descrip-

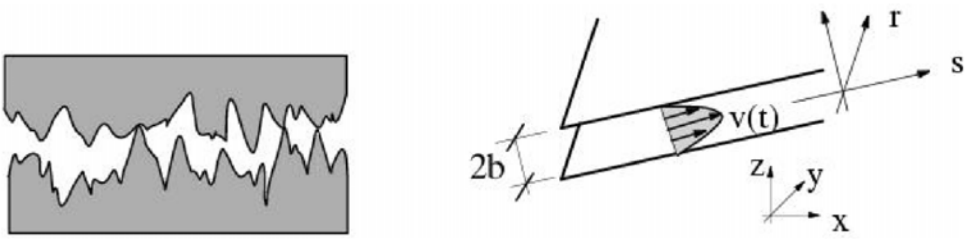

Figure 1 *Homogenisation of the local fracture scale*

tion of the coupled system for density-dependent groundwater flow and mass transport in saturated / unsaturated porous and fractured rock. In the presented numerical model (SPRING, 2010) the flow equation and the mass transport equation are coupled over a linear assumption.

In the numerical model we solve the decoupled flow and transport equation and couple them by an iteration. By using the Galerkin formulation and the Gauss-Green-Theorem, the flow and transport equations are transformed into integral equations, which only contain first derivatives of primary variables pressure and concentration. The resulting equations are given by Wendland (1996). Solution strategies for the high advective transport in fractures were introduced by König (1991) and Wendland (1996).

Generation of fractures

To generate fractures stochastically an ascertainment and evaluation of field data is necessary. Besides, the simulated fracture parameters are required to reflect the input data. At least the generated fractures have to go through a filtering process to prepare the data for mesh generation (König, 1998).

Figure 2 shows a fracture system with 1800 fractures which have been stochastically generated with the three different parameter sets of Table 1. The volume is 50 meters long, 25 meters wide and 15 meters high. The lengths of the generated fractures are lognormal distributed with the mean value 8.0 m and a standard deviation of 1.50 m. The apertures are lognormal distributed as well, while the mean value equals 250 mm. The standard deviation is unknown. It is assumed to be 30% to 50% of the mean aperature value (Wollrath, 1990).

Application

Numerical models are used to investigate long term safety of a disposal site in a mine. In addition to two-dimensional models of vertical cross sections, three-dimensional models lead to a better understanding for the physical processes. However, three-dimensional simulation of the entire

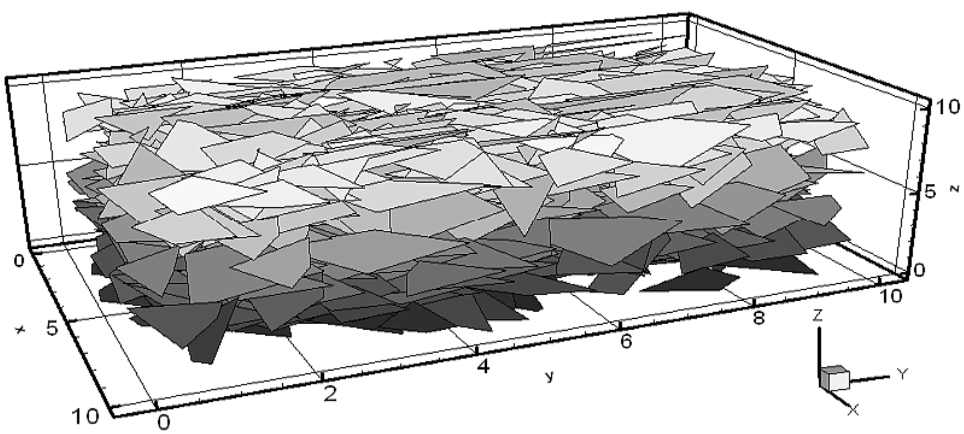

Figure 2 *Created fracture planes*

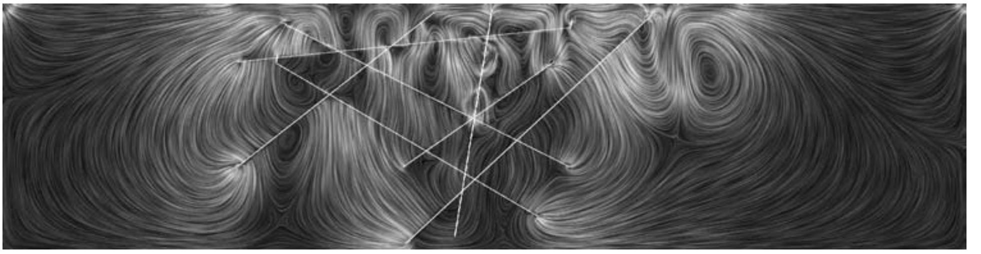

Figure 3 *Visualisation of the velocity field of the Elder-Problem with 1D-fractures (white lines)*

domain of interest exceeds available computational power. Therefore; in a first step a transient transport calculation in a three-dimensional subdomain was realized (König 1998).

At first a two-dimensional well known Elder-Problem with one-dimensional Fracture inside the domain. The boundary and starting conditions are the same as the Elder-Problem (Elder J.W. (1967). At starting time of the simulation the domain is filled with uncontaminated water (c = 0, δ = 1000 kg m^{-3}) which is in the steady state of hydrostatic pressure. At all boundaries of the domain no flow boundary conditions are assumed with the exception of the top left and right edge. At the top boundary a concentration of c = 1 is fixed with density of δ = 1200 kg m^{-3}. Figure 3 shows the concentration (grey-color).

In this section a density dependent flow and transport calculation for a three-dimensional subdomain of a fractured rock medium is presented. The three-dimensional model is a cube with edge lengths of 50 × 50 × 44 meters. According to the statistical input data approximately 20 fractures have been generated within this cube. The mesh generation in space as explained above resulted in 10 142 plane elements, 251 756 system nodes and 316 975 volume elements. The minimum number of element layers was set to 30 (Figure. 4).

The initial and boundary conditions for the test problem where chosen similar to the well know two-dimensional Elder problem (Elder, 1967).

At starting time of the simulation, the cube is filled with uncontaminated water (c = 0, δ = 1000 kg m-3) which is in steady state, hydrostatic pressure. At all boundaries of the cube, no flow boundary conditions are assumed with the exception of the top left and right edge (Figure 4). In an area of 24 × 5 meters at the top boundary, a concentration of c = 1 is fixed with density of δ = 1200 kg m^{-3}. The computing time for 50 timesteps for the density-dependent problem was about 1.5 h on a INTEL Core i7e @3.33GHz machine.

According to molecular diffusion, transport of the solute into the cube takes place. This leads to a modification of the fluid density inside the cube and hence a solute convection is developed.

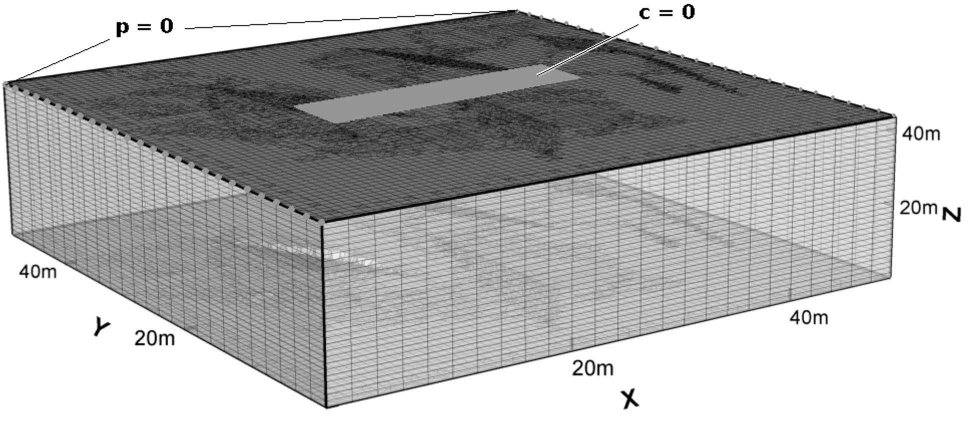

Figure 4 *Three-dimensional finite element model for a section in a sandstone layer*

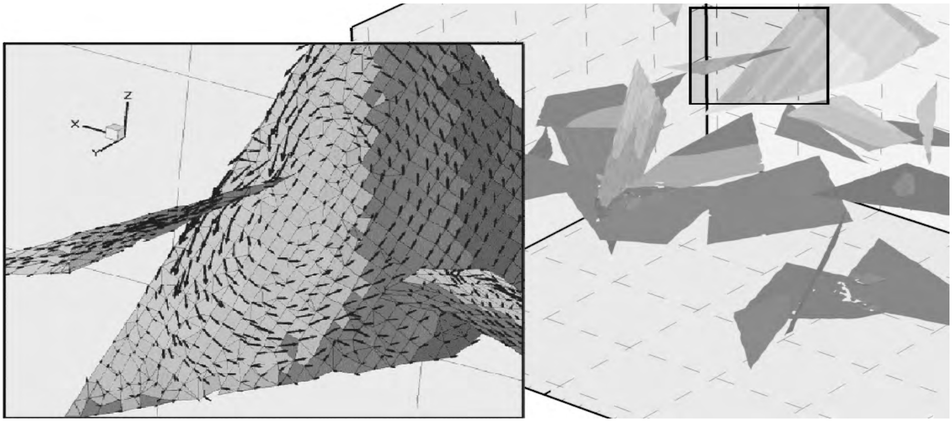

Figure 5 *Velocity vectors on the fracture after 50 timesteps, fractures coloured by c*

Figure 5 shows the velocity vectors of the fractures where convection cells can be seen equally. The typical convection cells can be seen which are disturbed by the discrete fractures.

Summary

A finite element model for calculation of three-dimensional density-dependent saturated and unsaturated flow and transport processes in coupled fractured porous media is presented. The fractures are generated by using statistical data and are approximated by plane elements. The volume elements of the surrounding porous rock matrix are generated by means of a layer technique.

The model is adopted to a three-dimensional problem with boundary conditions similar to the Elder problem. The results show the typical convection cells for solute convection problems and the influence of the discrete fractures. The approach shows that the coupled density depended calculation allows a better simulation of natural processes.

References

Elder J.W. (1967) Transient Convection in a Porous Medium. J. Fluid Mechanics 27.

Gambolati, G., Pini G. (1986) A 3-D finite element conjugate gradient model of subsurface flow with automatic mesh generation. Adv. Water Resources 9, 1486 – 1493.

König, C. (1991) Numerische Berechnung des dreidimensionalen Stofftransports im Grundwasser (Numerical calculation of threedimensional transport in groundwater). Ph.D. Thesis. Ruhr-Universität-Bochum.

König, C. (1998) Calculation and mesh generation of a three-dimensional matrix fracture system. J. Hydraulic Res. 36, 867 – 884.

SPRING User's Manual Version 3.4 (2010) delta h Ingenieurgesellschaft, Witten

Wendland, E. C. (1996) Numerische Simulation von Strömung und hochadvektivem Stofftransport in geklüftetem porösen Medium (Numerical simulation of flow and high advective transport in porous an fractured rocks). Ph.D. Thesis. Ruhr-Universität-Bochum.

Wollrath, J. (1990) Ein Strömungs- und Transportmodell für klüftiges Gestein und Untersuchungen zu homogenen Ersatzsystemen (A flow and transport model for fractured rock material and investigation of equivalent continuum substitute systems). PhD Thesis. Universität Hannover.