Application of the Null Space Monte Carlo Method in a Groundwater Flow Model of Mine Pit Dewatering

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Abstract
A groundwater flow model was developed to estimate groundwater inflow to a proposed open-pit gold and copper mine in a complex hydrogeologic setting with secondary porosity features (i.e. karst terrain). A sensitivity analysis, using a Null Space Monte Carlo technique, comparing mine pit inflow rates to changes in hydraulic conductivity (K) is presented where model calibration is maintained for the entire range of simulated pit inflows. The simulated range of inflows is 92 lps to 201 lps, with 131 lps median and 135 lps mean. Because the model remains calibrated, the entire simulated range of inflows can be employed for planning and preliminary engineering design. An additional advantage of this approach is that it identifies locations for further, or future, data collection to better refine pit inflow estimates.

Keywords groundwater, modeling, stochastic, dewatering

Introduction
MMG Limited completed a Primary Gold Prefeasibility Study (PFS) at the Sepon Gold and Copper Mine in the Lao People’s Democratic Republic (Laos). As part of the need for MMG to characterize the hydrologic and hydrogeologic conditions for mine expansion, substantial field characterization and hydrogeologic modeling works were completed (Ausenco, 2013). The project site encompasses an area of approximately 1,250 km² located 40 km north of the town of Sepon, Savannakhet Province, south-central Laos. The PFS hydrologic and hydrogeologic study considered numerous mine pits and hydrologic and hydrogeologic issues. However this paper describes the estimation of groundwater inflow to a single open pit, the Discovery West Pit (DSW-1) and a sensitivity analysis performed on hydraulic conductivity (K) within a highly heterogeneous dolomite unit to predict a range of groundwater inflow estimates.

Model Development
The Sepon groundwater model was formulated and simulated using MODFLOW-SURFACT (Hydrogeologic Inc., 2011). The model domain (fig.1) is a truncated representation of the Nam Kok watershed and represents an area of 200 km². Lateral model boundaries are simulated using General Head Boundaries (GHBs) at a sufficient distance from the area of interest to minimize boundary effects on the mine inflow solution. River boundary cells are used to represent major rivers, the Nam Kok and Nam Khieng Rivers and drain boundaries are used to represent tributaries. The calibrated recharge rate is 10 percent of measured site precipitation or 221.3 millimeters/year and was applied uniformly across the model surface. The model grid has variable horizontal spacing between 25 meters (m) and 525 m and 17 model layers for a total of 851,003 active cells.

The geologic model used in groundwater flow models is composed of hydrogeologic units (HGUs) representing rhyolite, dacite porphyry, dolomite, shale, a catchall “other” category and the two dolomite-karst zones identified from field data (fig.1). The areas of karstic dolomite are simulated as four zones, to represent different dolomite units, geographical areas, and depths, all of which affect the degree of karstification and measured K values.
Hydrogeological test results (i.e. pumping tests, slug tests, packer tests) suggest that karst secondary porosity features in two dolomite units (fig.1) have created zones of relatively high $K$ values (e.g. 102 m/d). However, the extent of dissolution is spatially variable; and in locations where solution features are filled with secondary clay or are absent, measured hydraulic conductivity values were on the order of 10^{-4} m/d. The conceptual approach to simulating groundwater flow in the karstic dolomite involves attempting to represent the discontinuous to partially continuous, relatively high hydraulic conductivity zones by estimating continuous fields of $K$ values (each cell has a unique or potentially unique $K$ value) for these zones in the groundwater model (fig.1).

Calibration methods and results

Groundwater levels in 171 piezometers were used as calibration targets. The model hydraulic conductivity values were varied (within the range of observed values) to obtain the best fit between simulated and measured groundwater levels using an inversion approach to achieve model calibration. The best fit is defined as the minimization of the sum of squared errors (SSE), where the SSE is the sum over all the data points of the square of the observed value less the simulated value (i.e. sum of squares of the residuals).

An advantage of the highly parameterized, regularization-assisted, inversion approach to calibration is that a Null Space Monte Carlo (NSMC) approach can be used to examine uncertainty and conduct sensitivity analyses on the hydraulic conductivity of the dolomite unit. For this project, a NSMC approach was used to generate 100 calibrated realizations of the hydraulic conductivity of the dolomite-karst unit in the model. Each realization was calibrated via highly parameterized inversion techniques (Watermark Numerical Computing, 2010). The calibration targets are primarily located in the vicinity of the mine pits and large regions of the model are relatively distant from the calibration target locations (fig.1). As a result, the inversion process (calibration) is relatively insensitive to changes in $K$ values in these regions that have few or no calibration target. Thus, these null space regions in the groundwater model can have a wide range of $K$ values without having a significant impact on
the model calibration statistics. This wide range of calibrated $K$ values in the insensitive regions of the model will, however, have an impact on the simulated inflows to the mine pits.

During steady-state model calibration, horizontal ($K_h$) and vertical ($K_v$) hydraulic conductivity values in the four dolomite-karst zones were allowed to vary across the range of hydraulic conductivity values collected as part of field investigations (0.0001 to 120 m/d). The model-calibration code PEST (Watermark Numerical Computing, 2010) was used to calibrate $K$ values using the pilot point technique (Doherty, 2003). An advantage of this approach is that it produces a continuously varying field of $K$ values where relatively high $K$ zones (karst areas) are adjacent to relatively low $K$ zones (e.g. areas without extensive solution weathering or karst areas with clay infill), thus capturing the high degree of spatially and vertically heterogeneous hydraulic conductivity measured in the field. Other model zones are calibrated with uniform values for $K_h$ and $K_v$.

The statistic employed by PEST to determine the optimal fit is minimization of the SSE. However, other statistical descriptors were also processed to evaluate the calibration of the Base Case model, including the following: 1) Residual Mean ($m$) = 0.19; 2) Absolute Residual Mean ($m$) 3.38; 3) SSE = 2.12E+3; and, 4) Normalized Root Mean Square Error (NRMSE) = 0.01. With a NRMSE of 1% the model was considered calibrated so that predictive simulations using the model would provide some insight into the possible response to dewatering of the DSW-1 Pit (fig.1).

**Predictive model and base case results**

The predictive simulations are transient simulations of pit development over time. For the transient simulations, storage parameters needed to be specified in addition to $K$ values. As a result, storage values were assigned as uniform values to the zones/regions shown in fig.1, based on a combination of values derived from pumping tests and literature values (for hydrogeologic units without pumping tests). While varying storage parameters and model boundary conditions (i.e. recharge) also affect pit dewatering rates, $K$ values are the most sensitive and the intent of this paper is to examine the impacts of changes in $K$ values using the NSMC approach.

Predictive pit dewatering simulations assume passive dewatering (i.e. sumps) and seepage into the various pit shells is simulated using drain boundary conditions. The predicted groundwater inflow rates from the calibrated model for years 1 through 4, of the four-year pit life, are 174 liters per second (lps), 126 lps, 155 lps and 108 lps, respectively; with a resulting average inflow rate of 131 lps. The changes in pit dewatering rates through time are a function of pit depths and pit sequencing; the removal of water from storage over time and the intersection of the pit floor and walls with saturated hydrogeologic units as the pit expands and deepens.

**Sensitivity analysis of the hydraulic conductivity of the karst zones**

Two types or categories of sensitivity analysis could be completed for the calibrated model. The first type is referred to here as the “traditional sensitivity analysis”, where the parameter of interest is varied either arithmetically or geometrically across a specified range and the resulting calibration and predictive simulation are then examined to determine the sensitivity of the solution to changes in that parameter (ASTM International, 2002). As discussed, this groundwater model has already been calibrated using an inversion procedure to obtain the best-fit parameter values. An arbitrary change to these values, that is multiplying each cell in the continuous field by a constant value ($\pm 25\%$) leads to an increase in SSE of over 3 orders of magnitude. A three order of magnitude increase in SSE produces a NRMSE value on the
order of 40%. As a result, the simulated $K$ distribution can no longer be considered calibrated because the $K$ field is unrealistic which renders corresponding estimates of pit inflow unreliable and demonstrates the limitations of this simple approach.

The second sensitivity approach, which is only available to models calibrated using a highly parameterized inversion approach, is a NSMC approach. The NSMC approach involves using a Monte Carlo simulation approach on the null space parameters (i.e. pilot points in portions of the model in which $K$ value changes will not impact the calibration). In a Monte Carlo simulation approach a number of simulations are conducted (100 in this case) by randomly changing a parameter of interest for each simulation. In the NSMC approach, the $K$ parameter values for pilot points in the null space, are set randomly (using the PEST utility within the range 0.1*calibrated $K$ to 10*calibrated $K$) for each Monte Carlo simulation. Each Monte Carlo simulation also involves a calibration to obtain a randomly selected but calibrated set of $K$ values in the null space. For each NSMC calibration, a threshold SSE value is specified so that the NSMC simulation is considered calibrated when the simulation SSE value is lower than the threshold value. In this case, the threshold SSE value was set to 5.0E+3, or a NRMSE of 2%. The NSMC approach then produces 100 calibrated realizations with a NRMSE of 2% or lower.

The NSMC approach provides a highly-parameterized, non-linear, post-calibration uncertainty analysis of model predictions of groundwater inflow to the proposed DSW-1 pit. Because this approach enforces parameter variations that produce a calibrated model; the range of inflows are all reasonable estimates with similar probabilities of occurring based on available data. The NSMC results predict a range of pit inflows from 92 lps to 201 lps with 25th and 75th percentile values of 117 lps and 150 lps, respectively. The estimated inflows were not normally distributed; having a positive skew towards the higher inflows (fig.2). The base case estimate of 131 lps was close to the average of 135 lps from the NSMC. The NSMC approach also identifies the areas of the model which do not contain sufficient data to constrain the $K$ values in the calibration. These are areas where additional data are required to constrain simulated pit inflows. These areas should be targeted for additional data collection.

![Fig. 2 Frequency histogram of NSMC pit inflow results](image)
Conclusions

This paper presents sensitivity analysis comparing mine pit inflow rates to changes in $K$ of a karstic dolomite zone. Application of the NMSC approach ensures that steady-state model calibration is maintained for all $K$ fields used in a transient simulation to predict a range of pit inflows. As a result, the entire range of simulated inflows resulting from uncertainty in $K$ distributions can be evaluated for planning and preliminary engineering design. One hundred equally-viable $K$ distributions (as determined from steady-state calibration statistics) result in a range of average inflows from 92 lps to 201 lps with a median of 131 lps and a mean of 135 lps. A further advantage of the sensitivity analysis approach presented here is that it identifies locations where future data collection efforts should be undertaken to improve the predictive capability of the model. The parameter uncertainty and sensitivity analysis presented here is an example of a Null Space Monte Carlo technique which relies on the use of highly parameterized inversion in combination with formal mathematical regularization techniques for model calibration. These advanced techniques can be implemented at PFS level of site investigation to provide a reasonable range of inflows on which to base preliminary dewatering system designs and costs, flows for water treatment systems and water management facilities and hydrologic impacts associated with project development.

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