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NUMERICAL MODELLING OF TWO PHASE FLOW OF GAS AND WATER DURING DRAINAGE OF A COAL SEAM

by

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ABSTRACT

This paper examines two phase flow of methane gas and water during drainage of a coal seam. A computer model is developed and applied to a mine at Bulli, N.S.W., Australia. Pressure and flow predictions for the two fluids, methane and water are made. For planners the point of interest is the drainage time required to achieve a desired seam gasiness and the most economical schedule of borehole drilling. That is, it is desirable to know the most effective borehole diameter, spacing and orientation, and the computer model can assist this planning.

Transient one dimensional flow in the coal seam is assumed. The coal seam is modelled as having isotropic coal properties such as porosity and permeability and the seam is assumed incompressible, isothermal and horizontal. The model assumes incompressible flow for water and ideal gas behaviour for methane. Flow through the coal seam is assumed laminar Darcy with full two phase capillary and relative permeability effects.

The flow equations are solved numerically using an explicit difference formulation bounded by experimentally determined initial conditions and a no flow condition at the axial limit, providing the ability to consider neighbouring borehole effects as desired. The output of the computer program is a graphical presentation of pressure distribution and flow rates at different times and at different locations within the coal seam.

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1.INTRODUCTION

Drainage of methane gas from a coal seam during mining operations has considerable significance for mine safety with regard to explosions. Methane drainage is achieved by drilling a set of drainage holes into the coal seam, and water, as well as gas, is drained. Effective drainage depends on the diameter, spacing and orientation of the drainage holes. In this paper, a mathematical model is developed for two phase flow of gas and water during drainage of a coal seam. The equations are solved numerically and the model is applied to experimental measurements made in mines operated by West Cliff Colliery at Bulli, New South Wales, Australia.

2. MATHEMATICAL MODEL FOR TWO PHASE FLOW

2.1 General

The mathematical model is developed assuming that the coal seam is isotropic with respect to porosity and that the seam is incompressible, isothermal and horizontal. The model assumes incompressible water and ideal gas behaviour for the methane. Ideal gas behaviour applies because temperatures in the coal seam are 22°C to 25°C, and maximum gas pressures are always less than 900 kPa.

2.2 Relative Permeability of the Coal Seam

Since two phase flow is being modelled, the absolute permeability of the coal seam k (which is an intrinsic property of the coal porosity and cleat network and independent of the fluids) is separated into two relative permeability components.

The fractional saturation of gas and water is denoted by sg and sw respectively where

$$0 \leq s_g \leq 1, 0\ s_w \leq 1 \text{ and } s_w + s_g = 1.0 \qquad (1)$$

The relative permeability of gas and water k_{rg} and k_{rw} respectively, are functions of the fractional saturation of each fluid and can be expressed in terms of s_w

Figure 1 shows the relation between relative permeabilities and s_w used in the computer model based on experimental data presented in [Ref. 7]. Note that for total saturation by water ($s_w = 1.0$) then $k_{rw} = 1$ and $K_{rg} = 0$, and conversely for total saturation by gas ($s_w = 0$) then $k_{rw} = 0$ and $k_{rg} = 1$.

2.3 Equations for Water and Gas

The basic equations describing two phase flow in porous media are the continuity equations for each phase:

$$\text{water: } \frac{\partial}{\partial x}(\rho_w u_w) = -\phi \frac{\partial}{\partial t}(\rho_w s_w) \qquad (2a)$$

$$\text{Gas: } \frac{\partial}{\partial x}(\rho_g u_g) = -\phi \frac{\partial}{\partial t}(\rho_g s_g) \qquad (2b)$$

Where
- ρ = fluid density
- u = fluid flux velocity
- s = fractional saturation
- ϕ = porosity of the coal bed
- x = distance from drainage hole
- t = time

Flow velocity is related to pressure gradient by Darcy's equation:

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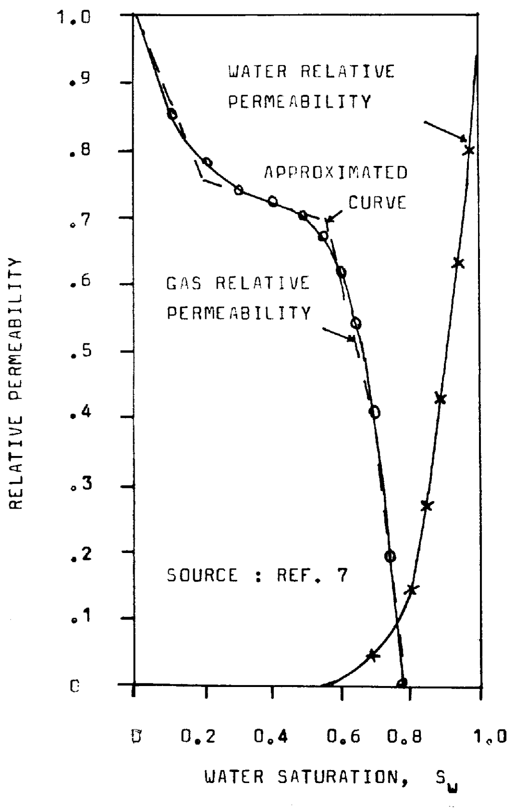

Figure 1 Gas and water relative-permeability curves used in computer model

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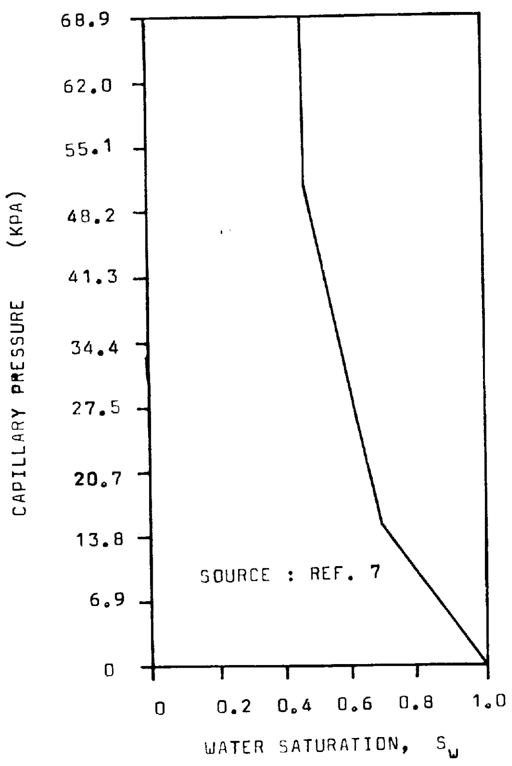

Figure 2 Capillary-pressure curve used in computer model.

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$$u_w = -\frac{k\,k_{rw}}{\mu_w}\frac{\partial p_w}{\partial x} \qquad (3a)$$

$$u_g = -\frac{k\,k_{rg}}{\mu_g}\frac{\partial p_g}{\partial x} \qquad (3b)$$

where k = absolute permeability
k_r = relative permeability
p = pressure
μ = fluid viscosity

Since water is considered incompressible, ρ_w is constant for a given temperature. Since ideal gas behaviour is adopted for methane, the gas equation

$$\rho_g = \frac{p_g}{RT} \qquad (3c)$$

applies,

where R = engineering gas constant for methane and
T = absolute temperature.

At the curved interface between methane gas and water a pressure difference, termed the capillary pressure p_c, exists due to surface tension effects.

$$p_c = p_g - p_w \qquad (3d)$$

As the water fractional saturation s_w decreases, more pore space is occupied by the methane gas and the gas/water interface retreats into smaller interstices of the pore space where radii of curvature are smaller. Consequently the capillary pressure p_c increases as s_w decreases.

Both p_g and p_w, and hence p_c are functions of (x,t). The fractional saturation of water s_w in the coal seam is also a function of (x, t) and the equations can be simplified by noting that a simple relation between p_c and s_w exists. Figure 2 shows the relation based on experimental measurements [Ref. 7] and this relation has been adopted in the mathematical model. Since $p_c = p_c(s_w)$ then the partial derivative

$\frac{\partial p_c}{\partial s_w}$ can be replaced by $\frac{dp_c}{ds_w}$.

2.4 Development of Model Equations

Combining equations (2a) and (3a) it can be shown that

$$\frac{dS_w}{dp_c}\frac{\partial p_c}{\partial t} = \frac{k\,k_{rw}}{\phi\mu_w}\left(\frac{\partial^2 p_w}{\partial x^2}\right) \qquad (4)$$

We apply the chain rule to $\frac{\partial S_w}{\partial t}$ because it is not possible to determine the rate of change of water saturation during drainage with time directly.

Using equation (3d) it can be seen that equation (4) is rewritten as:

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$$\frac{\partial p_g}{\partial t} - \frac{\partial p_w}{\partial t} = \frac{k\,k_{rw}}{\phi\mu_w}\,\frac{dp_c}{dS_w}\left(\frac{\partial^2 p_w}{\partial x^2}\right) \qquad (5)$$

Also, using equations (2b), (3b), (3d) and (3c), it can be shown that

$$\frac{k\,k_{rg}}{\phi\mu_g}\left(\frac{\partial p_g}{\partial x}\,\frac{\partial p_g}{\partial x} + p_g\,\frac{\partial^2 p_g}{\partial x^2}\right) = s_g\,\frac{\partial p_g}{\partial t} + p_g\,\frac{\partial s_g}{\partial t} \qquad (6)$$

As $s_g = 1 - s_w$, we have $\frac{ds_g}{dp_c} = \frac{-ds_w}{dp_c}$ (7)

$$\text{Also, } \frac{\partial s_g}{\partial t} = \frac{\partial s_g}{\partial p_c}\,\frac{\partial p_c}{\partial t}$$

$$= \frac{ds_g}{dp_c}\left(\frac{\partial p_g}{\partial t} - \frac{\partial p_w}{\partial t}\right), \text{ using equation (3d)} \qquad (8)$$

Substituting equation (7) and equation (8) into equation (6) we get

$$\left[\frac{1 - s_w}{p_g}\,\frac{dp_c}{ds_w} - 1\right]\frac{\partial p_g}{\partial t} + \frac{\partial p_w}{\partial t} = \frac{k\,k_{rg}}{2\phi\mu_g p_g}\,\frac{dp_c}{ds_w}\,\frac{\partial^2 p_g^2}{\partial x^2} \qquad (9)$$

Adding equations (5) and (9) we get

$$\left[\frac{1 - s_w}{p_g}\,\frac{dp_c}{ds_w}\right]\frac{\partial p_g}{\partial t} = \frac{k\,k_{rg}}{2\phi\mu_g p_g}\,\frac{dp_c}{dS_w}\,\frac{\partial^2 p_g^2}{\partial x^2} + \frac{k\,k_{rw}}{\phi\mu_w}\,\frac{dp_c}{ds_w}\,\frac{\partial^2 p_w}{\partial x^2}$$

$$\text{or, } \frac{\partial p_g}{\partial t} = \frac{k\,k_{rg}}{2\phi\mu_g\,(1-s_w)}\left[\frac{\partial^2 p^2 g}{\partial x^2}\right] + \frac{k\,k_{rw}\,p_g}{\phi\mu_w\,(1-s_w)}\left[\frac{\partial^2 p_w}{\partial x^2}\right] \qquad (10)$$

For simplification, assume $m = \frac{1 - s_w}{p_g}\,\frac{dp_c}{ds_w} - 1$

and Equation (9) and be written as

$$\frac{\partial p_g}{\partial t} + \frac{1}{m} \bullet \frac{\partial p_w}{\partial t} = \frac{k\,k_{rg}}{2\mu_g\,p_g\,m}\,\frac{dp_c}{ds_w}\,\frac{\partial^2 p_g^2}{\partial x^2} \qquad (11)$$

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Substracting equation (5) from equation (11)

$$\left(\frac{1}{m} + 1\right) \frac{\partial p_w}{\partial t} = \frac{k\, k_{rg}}{2\phi\mu_g\, p_g\, m} \frac{dp_c}{ds_w} \frac{\partial^2 p_g^2}{\partial x^2} - \frac{k\, k_{rw}}{\phi\mu_w} \frac{dp_c}{ds_w} \frac{\partial^2 p_w}{\partial x^2}$$

$$\text{or, } \frac{\partial p_w}{\partial t} = \frac{1}{1+m} \frac{k\, k_{rg}}{2\phi\mu_g\, p_g\, m} \frac{dp_c}{ds_w} \frac{\partial^2 p_g^2}{\partial x^2} - \frac{m}{m+1} \frac{k\, k_{rw}}{\phi\mu_w} \frac{dp_c}{ds_w} \frac{\partial^2 p_w}{\partial x^2}$$

$$\text{Also, } \frac{1}{1+m} \frac{1}{p_g} \frac{dp_c}{ds_w} = \frac{1}{\frac{1-s_w}{p_g} \frac{dp_c}{ds_w}} \frac{1}{p_g} \frac{dp_c}{ds_w} = \frac{1}{1-s_w}$$

Therefore, the above equation can be rewritten as

$$\frac{\partial p_w}{\partial t} = \frac{k\, k_{rg}}{2\phi\mu_g (1-s_w)} \frac{\partial^2 p_g^2}{\partial x^2} - \frac{m p_g k\, k_{rw}}{\phi\mu_w (1-s_w)} \frac{\partial^2 p_w}{\partial x^2} \qquad (12)$$

Equations (10) and (12) form a set of coupled equations which can be solved numerically to find p_g and p_w at all values (x, t).

2.5 Numerical Solution Method

Numerical methods are used to solve equations (10) and (12) to avoid the complexity involved in analytical techniques. A different equation method in explicit form is applied with known boundary conditions and an assumed initial pressure distribution.

Equations (10) and (12) are now replaced by explicit difference approximations:

$$p_g(x, t+\Delta t) = \frac{k\, k_{rg}}{2\phi\mu_g (1-s_w)} \frac{\Delta t}{\Delta x^2} \left[p_g^2(x+\Delta x, t) - 2p_g^2(x,t) + p_g^2(x-\Delta x,t)\right] + \frac{k\, k_{rw} p_g(x,t)}{\mu_w(1-s_w)} \frac{\Delta t}{\Delta x^2} \left[p_w(x+\Delta x, t) - 2p_w(x,t) + p_w(x-\Delta x,t)\right] + p_g(x,t) \qquad (13)$$

and

$$p_w(x,t+\Delta t) = \frac{k\, k_{rg}}{2\phi\mu_g (1-s_w)} \frac{\Delta t}{\Delta x^2} \left[p_g^2(x+\Delta x,t) - 2p_g^2(x,t) + p_g^2(x-\Delta x,t)\right] - \frac{k\, k_{rw} p_g(x,t)}{\mu_w(1-s_w)} \frac{\Delta t}{\Delta x^2} m \left[p_w(x+\Delta x,t) - 2p_w(x,t) + p_w(x-\Delta x,t)\right] + p_w(x,t) \qquad (14)$$

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Equations (13) and (14) are now used to calculate pressures p_g and p_w at time t + Dt from known conditions at time t. Values of s_w and k_r at times t and t + Dt are related to the values of p_g and p_w at these respective times, using the curves in the Figures 1 and 2 and the relation $p_c = p_g - p_w$. The numerical solution requires an iteration procedure in which estimates of s_w and k_r at time t + Dt are improved at each iteration. A separate sub-routine in the program is used for this.

3. APPLICATION OF MODEL

Data for West Cliff Colliery

Average temperature in mine entry	22°C
Water viscosity	1.0×10^{-3} Pa s
Gas viscosity	1.3×10^{-5} Pa s
Porosity	0.08
Absolute permeability	80 milli Darcys
Water density	1000 kg/m^3
Gas density	0.52 kg/m^3 at 80 kPa
Methane gas constant, R	518 J/kg.K
Gas-water relative permeability	Figure 1
Capillary pressure vs water saturation	Figure 2
Average pressure in mine entry	80 kPa

In the implementation of the explicit difference equation approximation to the differential equation describing fluid flow from the coal seam, the initial distribution is of major importance. As of subsequent pressure predictions and hence flow predictions are calculated from an original known pressure distribution it is important that an accurate distribution be known. Tests were conducted at West Cliff Colliery by Kembla Coal and Coke Pty. Ltd. [Ref. 5].

A 24 m deep drainage hole was constructed, and six 30 m deep parallel boreholes with pressure sensors were drilled along a line at distances 4, 6, 9, 16, 25, 35 m from the drainage hole. In each borehole pressure readings were taken at 0,10,20 and 30 m distances. Pressure readings obtained in this way are shown in Table 1.

Table 1

Initial Borehole Gas Pressure Readings Measured at West Cliff Colliery

Distance into	Distance of Borehole from Drainage Hole (m)						
Borehole (m)	0	4	6	9	16	25	35
0	80*#	80*	80*	80*	80*	80*	80*
10	80#	-	-	93	96	99	108
20	80#	140	-	290	348	-	359
30	80#	854	854	860	878	892	887

*Pressures in mine entry. #Pressures in drainage hole

Note: Values are absolute pressures in kPa.

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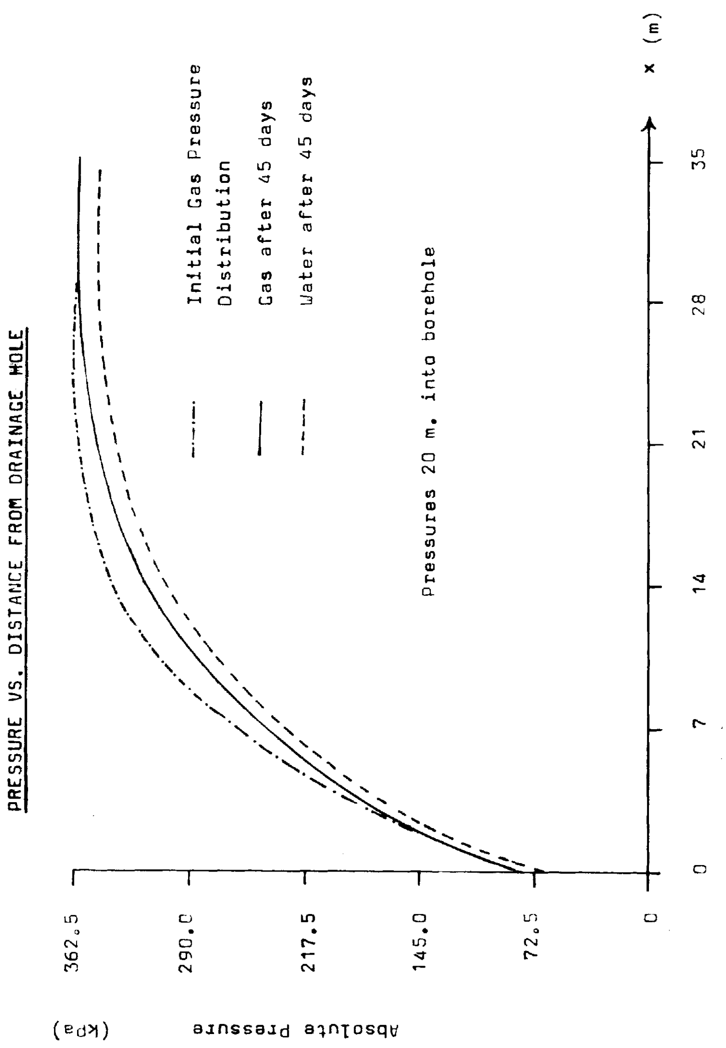

Figure 3 Pressure Distribution after 45 days

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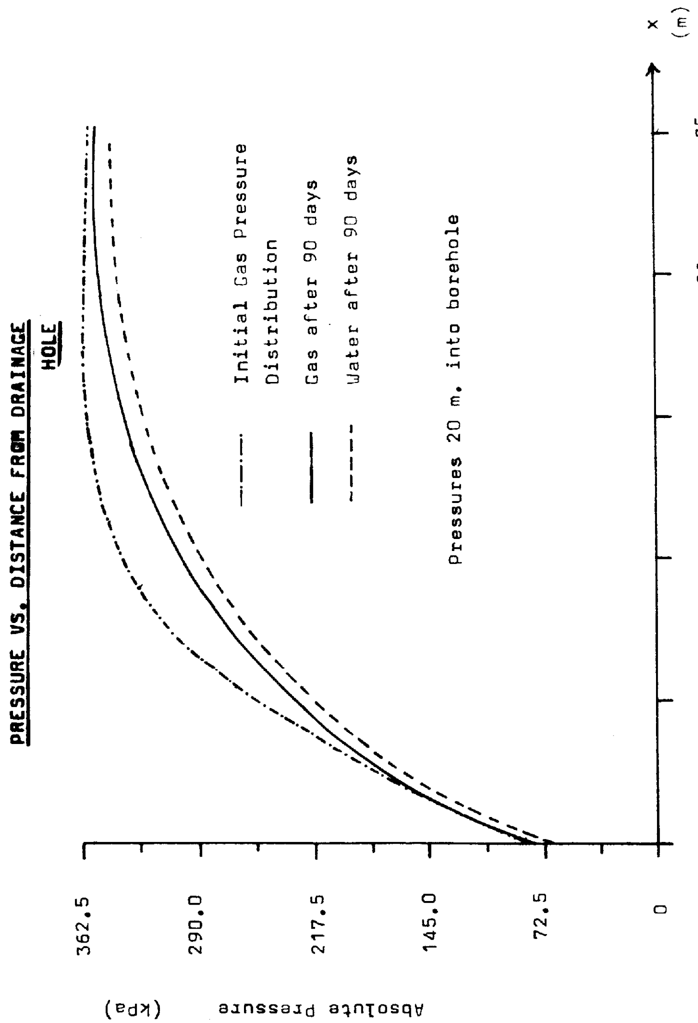

Figure 4 Pressure Distribution after 90 days

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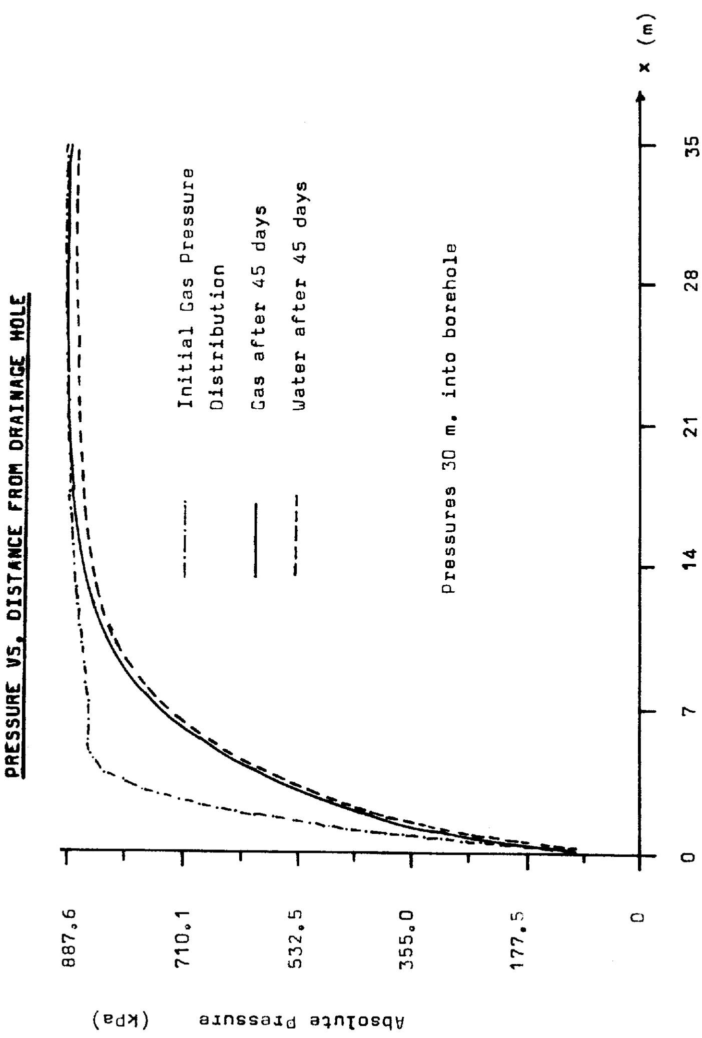

Figure 5 Pressure Distribution after 45 days

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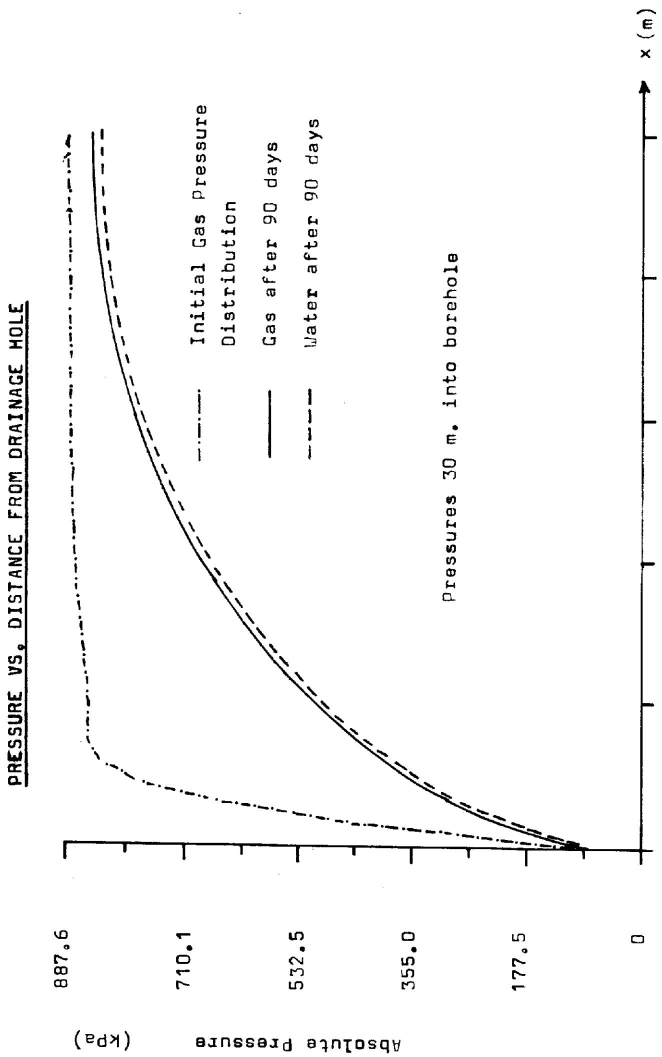

Figure 6 Pressure Distribution after 90 days

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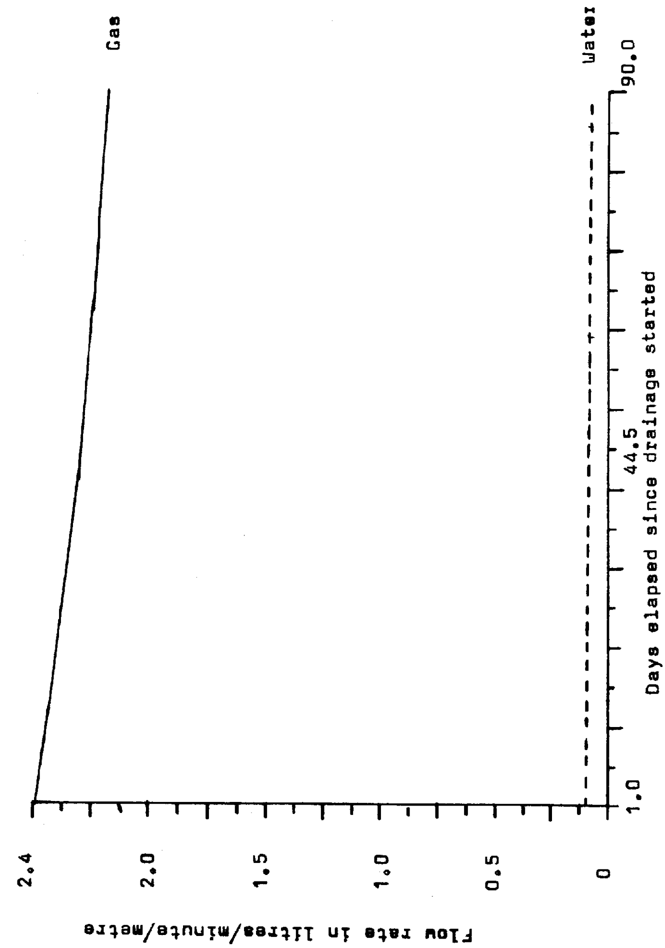

Figure 7 Flow rate history for 90 days

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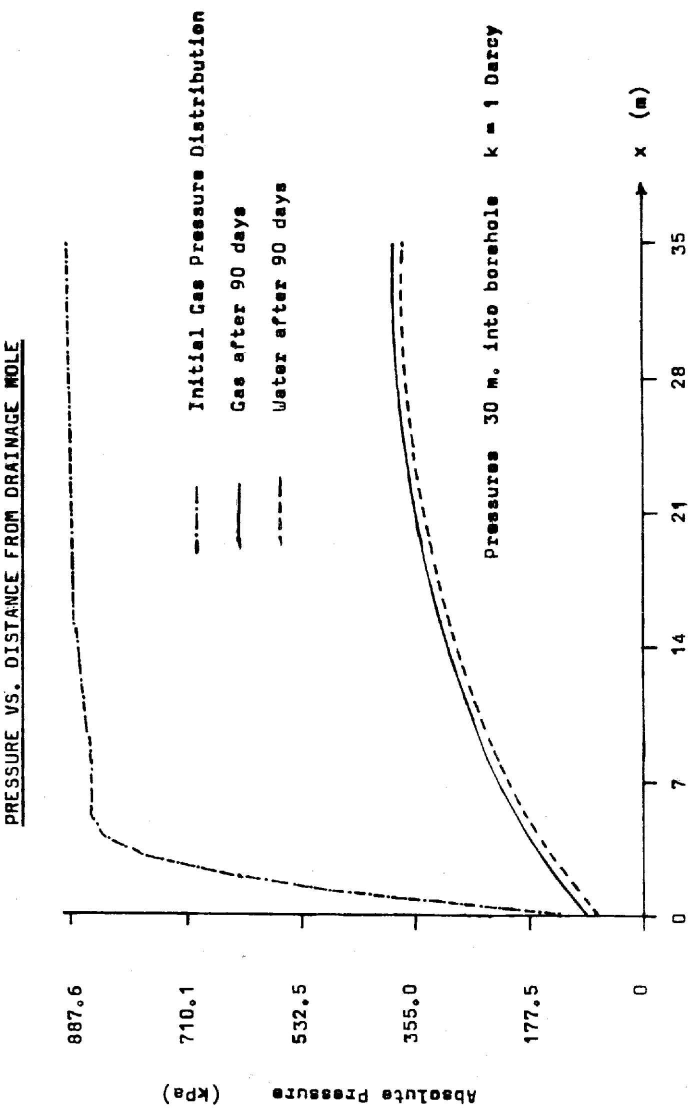

Figure 8 Pressure Distribution after 90 days

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Pressures at all points in the coal seam were interpolated from Table 1 using a cubic spline curve.

Permeability in coal seams can vary widely, from 50 milli Darcy to 1 Darcy. Accurate values of permeability have not yet been determined for Bulli mines, however a value of 80 milli Darcy has been suggested by Kembla Coal and Coke [R.D. Lama, personal communication]. All calculations are based on permeability of 80 milli Darcy. The effect of assuming a different permeability (1 Darcy) was also studied using the numerical model.

The following boundary conditions at the borehole (x = 0, t = 0) are used:

p_g = 80 kPa
s_w = 0.763
p_c = 12 kPa
p_w = 68 kPa

4. RESULTS OF COMPUTER MODELLING

Figures 3 to 6 show pressure distributions of gas and water within the coal seam, calculated at times 45 and 90 days after drainage commences using the parameter values given in Section 3.

Pressures generally decrease as flow drains towards the drainage hole. At all points in the coal seam, pressures decrease with elapsed drainage time. This is more noticeable at points 30 m into the boreholes where the pressure gradient is steeper.

For distances in excess of 35 m from the drainage hole, pressure of both gas and water remain close to initial values and do not decrease with time.

Figure 7 shows flowrates of gas and water per metre of borehole length, calculated over a 90 days period. Flowrates of water remain essentially constant with time and this agrees with experimental measurements taken at Bulli mine by Kembla Coal and Coke Pty. Ltd. [Ref. 5]. The flowrate of methane gas decreases slightly with increasing time as the gas content of the coal seam is depleted. The maximum value of 2.4 litres/minute/metre length of borehole agrees reasonably well with experimental values of 1.96 litres/minute/metre measured in Bulli mines [Ref. 5].

As stated in Section 3, permeability can vary widely in coal seams. figure 8 shows results calculated using a permeability of 1 Darcy. Comparing Figures 6 and 8, the increased permeability increases drainage of gas and water and produces a significant decrease in pressure, even at distances well away from the drainage hole.

5. CONCLUSIONS

A computer model has been developed for two phase flow of gas and water during drainage of a coal seam. The model is based on solution of the continuity equation and Darcy equation, applied to each phase. The resulting coupled equations are solved in an explicit finite difference formulation.

The computer model has been applied to experimental measurements taken at Bulli Mine, N.S.W. Australia and agreement between predicted and experimental pressures and flowrates is generally good.

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