Atkinson, L. C. & Gale, Y. E. (1985): Minimizing Well losses in Dewatering Wells in fractured Rock. – Proceedings, 2nd International Mine Water Association Congress, 1: 69-81, 8 fig.; Granada.

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The total drawdown or head loss induced in a well by pumping can have as many as four components (Figure 1):

$$s = s_1 + s_n + s_e + s_w \tag{1}$$

where

s = total head loss or drawdown,
s_1 = linear aquifer head loss or energy lost in overcoming viscous drag as ground water moves through the formation under low velocity, laminar conditions,
s_n = non-linear aquifer head loss or energy lost during flow through the high velocity, turbulent region which can develop in the immediate vicinity of the well,
s_e = exit loss resulting from water moving from the aquifer into the wellbore, and
s_w = wellbore loss incurred during flow in the wellbore to the pump intake.

Incorporating the concept of critical radius, defined as the distance from the center of the well to the point in the formation where the transition from linear to non-linear flow occurs, a more quantitative version of Equation 1 can be derived (Atkinson, 1985) as

$$s = \frac{Q}{2\pi T_1} \ln \frac{r_o}{r_c} + \frac{Q^2}{(2\pi T_n)^2} \left[\frac{1}{r_w} - \frac{1}{r_c} \right] + C_e Q^2 + C_w Q^2 \tag{2}$$

where

C_e = exit loss coefficient,
C_w = wellbore flow loss coefficient,
Q = volumetric discharge rate,
r_c = critical radius,
r_o = distance from center of well to point of negligible drawdown (radius of influence),
r_w = radius of the wellbore,
T_1 = transmissivity under laminar flow conditions, and
T_n = transmissivity under non-linear flow conditions

in any set of dimensionally consistent units. The wellbore loss, exit loss, and the excess drawdown in the formation (relative to that which would occur if linear flow conditions were to prevail to the radius of the well) due to the occurence of non-linear flow comprise what is collectively referred to as well loss.

In most dewatering systems using wells, it is normally desirable to achieve the required drawdown with the minimum number of wells and size of pumps. Usually this involves trying to pump each well at or near its maximum possible rate. Figure 2 schematically indicates how well losses comprise a non-beneficial drawdown which can hinder these objectives. The drawdown in the formation beyond the critical radius is, for the given hydraulic properties of the formation, a function of discharge rate. Since discharge is limited by the available drawdown in each well, any well loss will result in the discharge being less than the maximum attainable. Consequently, less drawdown is propagated into the formation by the wells. This means that more wells at closer spacings are required to achieve the same composite drawdown as that which would be produced by fewer wells not experiencing such losses. Another inefficiency introduced into the system by well loss is that, for a given discharge rate, the pumping lift is unnecessarily high. Larger, more expensive pumps (both with respect to capital and operating costs) could be required.

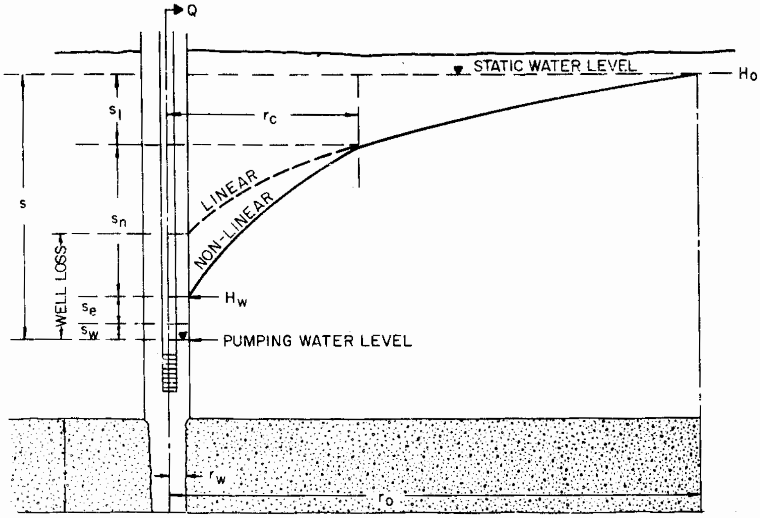


Figure 1. Components of drawdown in a well.

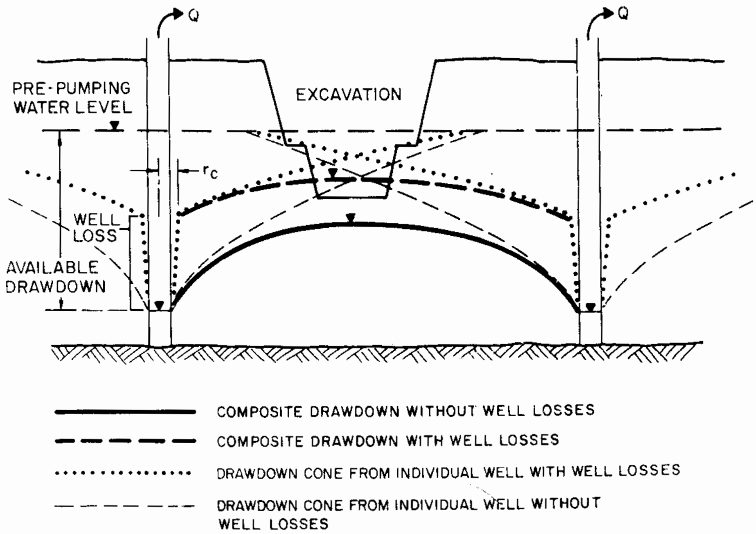


Figure 2. Schematic diagram of two-well dewatering system showing effects of well losses.

The traditional method for evaluating the performance of a well and attempting to quantify well loss is by means of a multi-rate or step-drawdown test (Brereton, 1979). The results of this test are usually expressed in the form

$$s = BQ + CQ^n \tag{3}$$

where

B = aquifer coefficient,
C = well loss coefficient, and
n = an exponent.

The exponent is usually considered to be 2, but its value has been the subject of some controversy. Comparison of Equation 3 with Equation 2 suggests they are equivalent, presuming n = 2 and the last three terms in Equation 2 containing Q^2 can be combined into a single term. It has been found that Equation 3 with its empirically determinable coefficients (and exponent, if other than 2) yields reasonable estimates of the drawdown which can be expected in a well at various pumping rates, provided they are within the range tested. This purely empirical approach, however, provides little information on the nature and causes of the well loss.

Although well loss can occur in almost any pumping well, the problem appears to be much more acute in formations where the primary source of permeability is fractures as opposed to intergranular porosity (Caswell, 1985; Mackie, 1982; Brereton, 1979; Uhl et al., 1976). Since mine dewatering wells frequently are completed into such units, attention is focused specifically on the problem of well losses in wells completed into fractured rock. In this study, current concepts of fracture rock hydrology (a comprehensive review is given by Gale, 1982) are applied in an attempt to develop a more deterministic model for the response of such a well to pumping. In other words, transmissivities, coefficients, and boundary conditions based on empirically derived laws for flow through rough fractures are incorporated into the relationship expressed in Equation 2. Using this more deterministic model, the potential effects on minimizing well losses of two wellbore stimulation techniques -- enlarging the wellbore and hydraulic propping -- can be evaluated.

THEORY OF TWO-REGIME, CONVERGENT RADIAL FLOW TO A WELL FROM A HORIZONTAL FRACTURE

In this development of a theoretical description of radial flow to a well, a single, horizontal fracture will be considered. Statistically, vertical wells (and most dewatering wells are vertical) primarily will encounter low angle fractures. Rissler (1978) demonstrated with a numerical model that the effects of the angle of intersection between the fracture and the wellbore are not significant until it becomes greater than about 50 degrees. Limiting attention to a single fracture also is not unrealistic. Several investigators (Da Cruz and De Quadros, 1984; Williamson and Woolley, 1980; and Baker, 1955) have concluded, based on the results of field data, that one fracture often totally dominates the production or injection rate of a well.

The earliest analyses of flow through fractures used the highly idealized model of laminar flow between smooth, parallel surfaces. An exact mathematical description of such flow -- known as the Navier-Stokes equation -- yields, when combined with the continuity equation, the so-called "cubic flow law":

$$Q_w = \frac{g(2b)^3}{12\nu}\frac{dh}{dl} \tag{4}$$

where

Q_w = volumetric flow rate per unit width normal to the direction of flow,
g = gravitational acceleration,
$2b$ = aperture of the fracture,
ν = kinematic viscosity of the fluid, and
dh/dl = hydraulic gradient.

Obviously, real fractures under field conditions vary significantly from such an idealized physical model. Attempts to develop basic laws for flow in more natural, rough fractures have utilized laboratory models of artificial fractures and application of the traditional friction factor (λ) versus Reynolds number (Re) relationship for characterizing flow through pipes. Two of the most comprehensive studies, including both laminar and non-linear flow conditions, were those of Lomize (1951) and Louis (1969). Comparisons and shortcommings of these and other developments of empirical flow laws are discussed in Atkinson (1985), Gale et al. (1985), and Pearce and Murphy (1979).

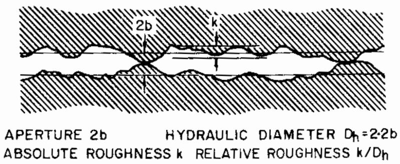


Figure 3. Geometric properties of a fracture.

Flow laws based on Louis' (1969) experimental work have been used in this study for two reasons: 1) the flow fields (using the λ versus Re relationship) in which the laws are applicable have been defined relatively well and 2) Rissler (1978) indicates a good correlation between theoretical results derived from these laws and his experimental data from a two-regime (i.e., including both regions of laminar and non-linear flow), divergent, radial flow model. The geometric properties of the fracture used to characterize the flow are defined in Figure 3 and friction factors and derived values of single-fracture transmissivity based on Louis' (1969) findings are given in Table 1.

A most important factor in Equation 2, but one with considerable uncertainty (Pearce and Murphy, 1979), is the critical radius. Louis (1969) found that up to a relative roughness of 0.0168, the critical Reynolds number, Re (the Reynolds number at which flow in a fracture becomes non-linear), is essentially constant and equal to approximately 2300. The boundary between smooth, transitional and smooth, fully turbulent flow was found to be function of roughness, as were the boundaries between flow regimes at roughnesses greater than 0.0168. By solving the various friction factor relationships simultaneously, Rissler (1978) derived the equations for the boundaries of Louis' flow fields given in Table 2.

From the basic definition of Reynolds number,

$$Re = \frac{vD_h}{\nu} = \frac{QD_h}{A\nu} = \frac{Q(2\cdot 2b)}{2\pi r(2b)\nu} \tag{5}$$

where

A = cross-sectional area of flow,
D_h = hydraulic diameter, and
v = average velocity of fluid,

Table 1. Friction factors for fracture flow and derived relationships for transmissivity

Regime		Roughness	Law	λ in Darcy-Weisbach[2] Equation	Transmissivity (T) in Thiem-like Equation[3]
Linear (Laminar)		Smooth	1	$\frac{96}{Re}$	$\frac{g(2b)^3}{12\nu}$
Linear (Laminar)		Rough	4	$\frac{96}{Re}\left[1+8.8(k/D_h)^{1.5}\right]$	$\frac{g(2b)^3}{12\nu[1+8.8(k/D_h)]^{1.5}}$
Non-Linear	Trans.	Smooth	2	$0.316 \cdot Re^{-1/4}$	$5.20\left[\frac{g^4(2b)^{12}}{(dh/dl)^3}\right]^{1/7}$
Non-Linear	Fully Turbulent	Smooth	3	$\left[2 \cdot \log\left(\frac{3.7}{k/D_h}\right)\right]^{-2}$	$4 \cdot \log\left(\frac{3.7}{k/D_h}\right)\left[\frac{g(2b)^3}{dh/dl}\right]^{1/2}$
Non-Linear	Fully Turbulent	Rough	5	$\left[2 \cdot \log\left(\frac{1.9}{k/D_h}\right)\right]^{-2}$	$4 \cdot \log\left(\frac{1.9}{k/D_h}\right)\left[\frac{g(2b)^3}{dh/dl}\right]^{1/2}$

Notes: 1. Smooth: $k/D_h < 0.033$; Rough: $k/D_h > 0.033$

2. $\frac{dh}{dl} = \lambda \frac{1}{D_h} \frac{v^2}{2g}$

3. $s = \frac{Q}{2\pi T} \ln(r_2/r_1)$ for Laws 1 and 4

$s = \left[\frac{Q}{2\pi T}\right]^{7/4} \left[\frac{1}{r_2{}^{3/4}} - \frac{1}{r_1{}^{3/4}}\right]$ for Law 2

$s = \left[\frac{Q}{2\pi T}\right]^{2} \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$ for Laws 3 and 5

Table 2. Critical Reynolds numbers

Law Boundary	Applicable Range of k/D_h	Re_c
1-2	<0.0168	2300
2-3	<0.0168	$2.552\left(\log \frac{3.7}{k/D_h}\right)^8$
1-3	$0.0168 \leq \frac{k}{D_h} \leq 0.033$	$\left[142000\left(\log \frac{3.7}{k/D_h}\right)^2\right]^{0.568}$
4-5	>0.033	$\left[142000\left(\log \frac{1.9}{k/D_h}\right)^2\right]^{0.568}$

a value of critical radius can be estimated by

$$r_c = \frac{Q}{\pi \nu Re_c} \; . \tag{6}$$

This indicates that, except for very smooth fractures (k/D_h <0.0168), the critical radius is a function of kinematic viscosity, discharge rate, and, as indicated in Table 2, relative roughness.

The last two components of drawdown in Equation 2 are relatively minor and need not be discussed in great detail. Using a plastic parallel plate model with air for a testing fluid, Murphy and Pearce (1980) investigated the nature and magnitude of exit loss, the pressure loss incurred during the transition from radial convergent flow in a fracture to longitudinal flow within an orthogonal wellbore. An empirical equation for this loss derived from their data during the current investigation is

$$s_e = 0.23\left(\frac{D_w}{2b}\right)^{1.41}\frac{v_w^{\,2}}{2g} \tag{7}$$

where

D_w = diameter of the wellbore, and
v_w = velocity of the fluid in the wellbore.

The head loss due to flow in the wellbore is the easiest component to quantify because of its direct analogy to pipe flow for which the classic Darcy-Weisbach equation for flow in circular conduits is applicable.

Using the values for fracture transmissivity given in Table 1, values for critical radius from Equation 6 and Table 2, and the empirically derived value for the exit loss coefficient, a quasi-deterministic version of Equation 2 can be obtained. The term "quasi-deterministic" is appropriate in that although every attempt has been made to include basic physical relationships for flow, the flow laws themselves and the range (defined by roughness and Reynolds number) over which they are applicable are completely empirical. For a relative roughness greater than 0.033 (and most natural fractures exceed this roughness), Equation 2 can be written as

$$\begin{aligned} s = {} & \left[\frac{6Q\nu\{1+8.8[k/(2\cdot 2b)]^{1.5}\}}{\pi g(2b)^3}\right] x \\ & \left[\ln\frac{r_o \nu \pi}{Q}\left(142000[\log\{1.9/[k/(2\cdot 2b)]\}]^2\right)^{0.568}\right] \\ & + \left[\frac{Q^2}{64\pi^2 g(2b)^3[\log\{1.9/[k/2\cdot 2b)]\}]^2}\right] x \\ & \left[\frac{1}{r_w} - \frac{\nu\pi(142000[\log\{1.9/[k/(2\cdot 2b)]\}]^2)^{0.568}}{Q}\right] \\ & + 0.23\left(\frac{2r_w}{2b}\right)^{1.41}\frac{Q^2}{2g\pi^2 r_w^{\,4}} + f_w\frac{(L-10r_w)Q^2}{4\pi^2 g r_w^{\,5}} \end{aligned} \tag{8}$$

where

f_w = wellbore friction factor and
L = distance between the fracture and the pump intake.

APPLICATIONS TO MINIMIZING WELL LOSSES

Recognition of Non-Linear Flow in the Formation

Mackie (1982) describes the results of more than 20 carefully controlled step-drawdown tests of wells completed in fractured rock and concludes that most of them display one of three "signature reponses." These responses are best depicted on a graph of specific drawdown, s/Q, versus discharge rate (Figure 4). Dividing both sides of Equation 3 (assuming n = 2) by the discharge rate, it can be shown that

$$s/Q = B + CQ. \tag{9}$$

The straight, horizontal line (Curve 1) represents tests during which all flow is laminar. For this case, the right hand side of Equation 9 is simply equal to B (presumed to be a constant). Curve 2 indicates the production-drawdown response when, up to the critical discharge rate (Q_c), all flow is laminar; but beyond that rate, head loss due to fully turbulent flow comprises an ever increasing portion of the drawdown. The total specific drawdown is composed of a constant component of value B and, beyond the critical discharge rate, a component linearly proportional (by the value C) to the discharge rate. Mackie (1982) states that in many tests, only the latter portion of the curve is evident, indicating non-linear flow even at relatively low flow rates.

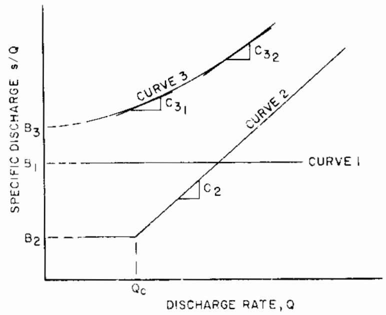


Figure 4. Typical production-drawdown responses of wells in fractured rock (after Mackie, (1982).

The response which Mackie (1982) states is most typical for wells in fractured rock is depicted by the concave upward curve (Curve 3) which might or might not have a short linear (either horizontal or sloping) segment at the lower production rates. This suggests the value for C in Equation 3 is not constant, which is contrary to the traditional assumption.

Using Equation 8, a step-drawdown test can be synthesized. The result, shown in Figure 5 (the exit and wellbore losses for this case are too small to be discernible at the given scale), at the given scale), is similar to Curve 3 of Figure 4. Variation in the value of C with discharge is predictable by comparing Equation 8 with Equation 3. This suggests that the "signature" represented by Curve 3 is indicative of wells in which non-linear flow in the formation is the main cause of well loss.

Evaluation of Potential Well Stimulation Procedures

The potential effects on minimizing well losses in dewatering wells in fractured rock of two well stimulation procedures -- enlarging the well-

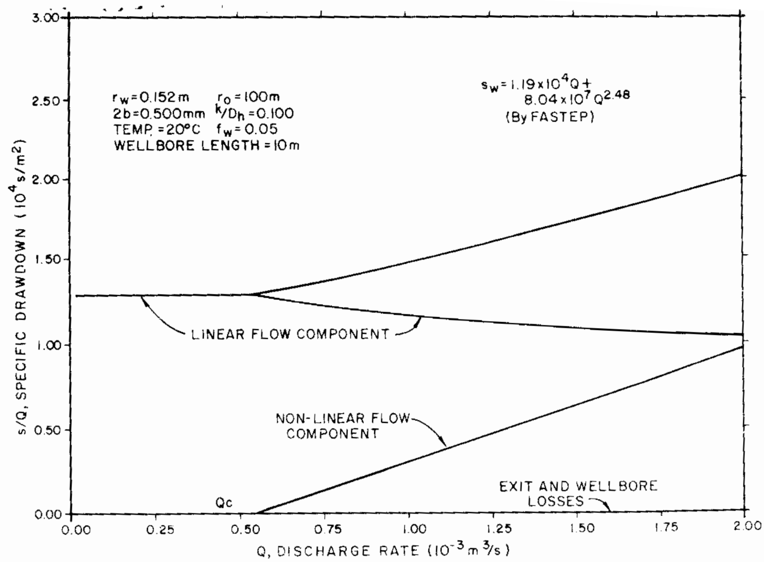


Figure 5. Specific drawdown versus discharge rate for a synthesized step-drawdown test.

bore and hydraulic propping of existing fractures -- can be evaluated using Equation 8. As part of this study, however, a more versatile numerical model (code named DEFLOW) based on the finite element method was developed. This numerical model has the flexibility of being able to handle both first (head) and second (discharge) type boundary conditions at the well whereas the analytical solution (Equation 8) is limited to second type boundary conditions. More importantly, the numerical model is a coupled model including a code to estimate deformation in the fracture resulting from changes in flow-induced changes in effective stress. Details of the numerical model are given in Atkinson (1985).

Figure 6 shows the wellbore radius versus production rate relationship for a theoretical well intersecting a single, horizontal fracture both as calculated by DEFLOW and as predicted by the classical $\ln(r_o/r_w)$ response. Based on the latter, the relative increase in yield in doubling the wellbore radius, for example from 0.1 m to 0.2 m, is approximately 10 percent whereas DEFLOW predicts an increase of 33 percent. This suggests that enlarging the wellbore or selectively under-reaming the wellbore in the producing interval could be an effective way of significantly increasing the production of the well and, therefore, increasing the drawdown which it can propagate into the formation. This supports and gives a deterministic basis to similar, earlier conclusions by Caswell (1985), Norris (1976), and Baker (1955).

Another method which has been used to stimulate wellbores in fractured rock is by hydraulic propping (Williamson, 1982; Williamson and Woolley, 1980). Hydraulic propping is differentiated from hydraulic fracturing in that in the former, existing fractures are widened by injection of fluid under excess pressure (relative to normal formation pressure) with or

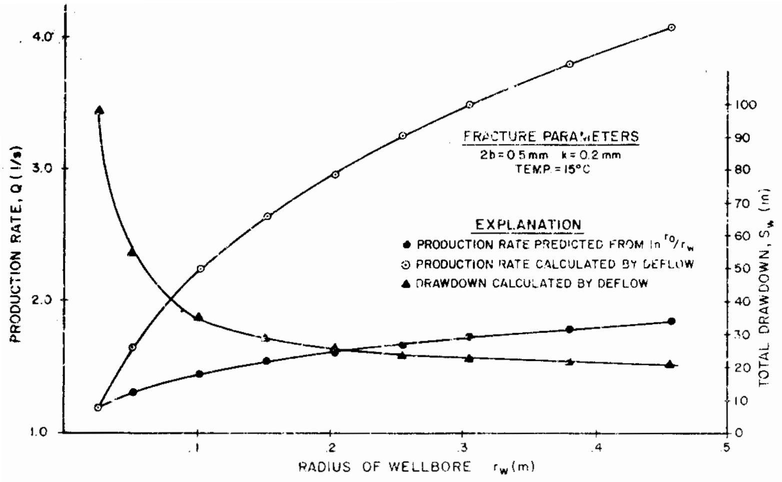


Figure 6. Effect of wellbore size on pumping response of a well intersecting a single, horizontal fracture.

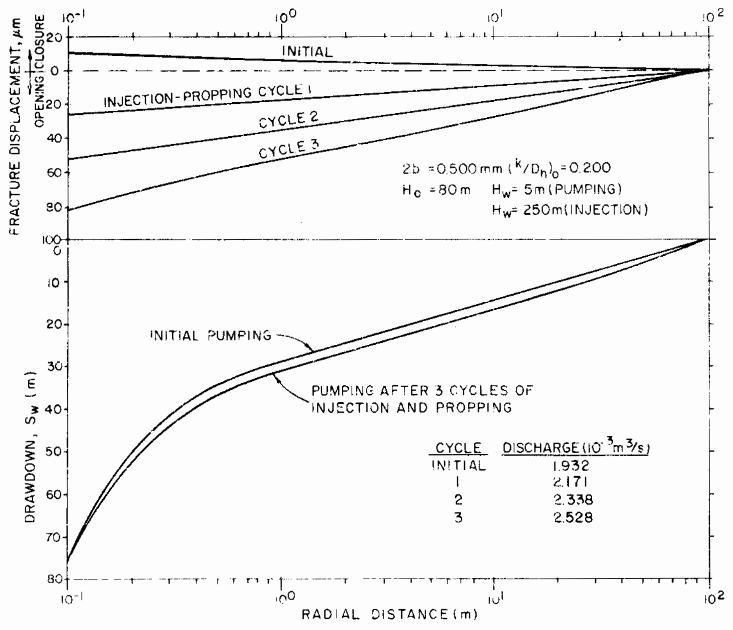
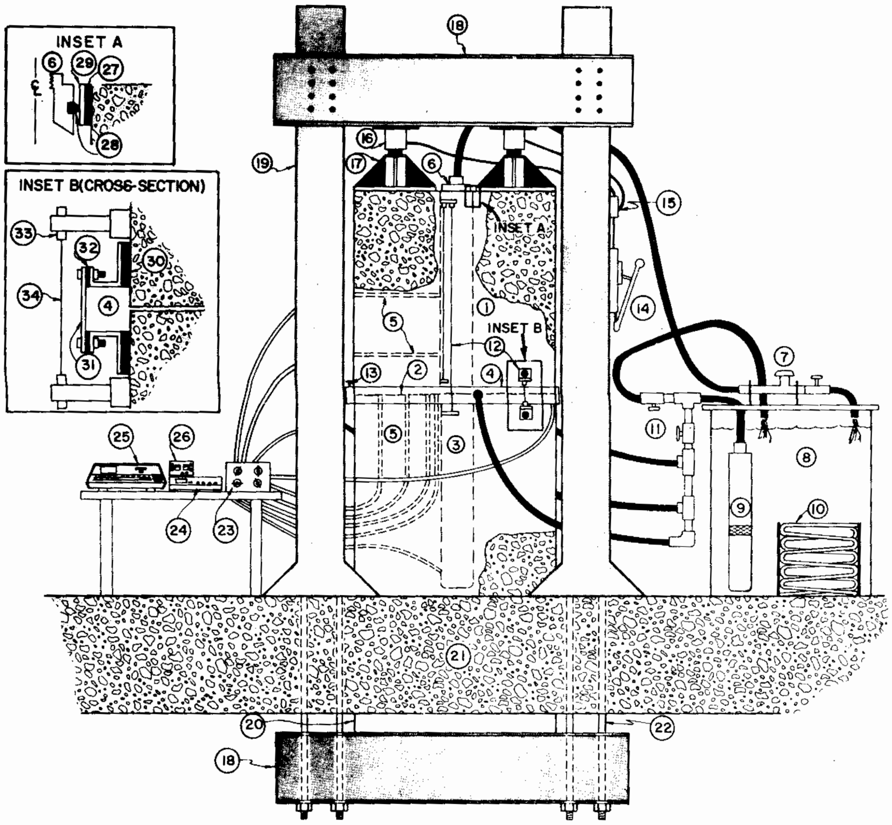


Figure 7. Potential effects of hydraulic propping on production-drawdown response.



1) Concrete cylinder (1.50 m diameter by 3.02 m high)
2) Cast fracture
3) Wellbore (0.064, 0.108, and 0.160 m diameter)
4) Circumferential reservoir
5) Manometers (5 in wellbore, 23 in fracture, 3 in circumferential reservoir)
6) Well collar
7) Paddlewheel flowmeter
8) Water supply tank
9) Submersible pump
10) Thermostat-controlled refrigerant unit
11) Pressure regulating valves
12) Linear variable displacement transformer (LVDT) assembly
13) Thermocouple
14) Hydraulic jack
15) Pressure manifold
16) Hydraulic cylinder (4 with 90.7 metric ton load each)
17) Load distributor
18) Channel beam
19) H beam
20) Box beam
21) Reinforced concrete floor (0.76 m thick)
22) Steel bolts (51 mm diameter)
23) Manometer-to-transducer multiplexer
24) Data acquisition and control unit
25) Personal computer
26) Power supply
27) Structural cement
28) O-ring
29) Brass ring
30) Poured-in-place polyurethane gasket
31) Steel band
32) Rubber gasket
33) LVDT
34) Invar rod

Figure 8. Radial flow model at Memorial University

without propping additives (sand, plastic beads, etc.). The results of attempts to stimulate water wells by this technique have been quite variable (Williamson, 1982). Using the coupled deformation-flow codes of DEFLOW, hydraulic propping of an existing 0.5 mm fracture was simulated. The results, shown in Figure 7, for a hypothetical field condition indicate that a 12 percent increase in yield is achieved as the result of opening the fracture in the immediate vicinity of the wellbore on the order of 20 μm. Subsequent cycles of injection (assuming the fracture remains at its new aperture and residual stresses are "relaxed" by closure in other fractures between cycles) show increases of 7 or 8 percent.

ADDITIONAL INVESTIGATIONS

Recent studies (Gale et al., 1985; Pearce and Murphy, 1979) have questioned the universal applicability of existing theoretical fracture flow models over the range of natural fracture conditions (e.g., types and scale of roughness, open fractures with no contact between adjacent walls versus fractures with considerable contact area). In order to evaluate the validity of the mathematical model developed in this study, a laboratory investigation utilizing a large scale, radial flow model (Figure 8) also was undertaken. Future publications will discuss the level of agreement between the theoretical and laboratory results.

SUMMARY AND CONCLUSIONS

- Excess drawdown due to non-linear flow in the formation in the immediate vicinity of a wellbore tapping horizontal fractures can be the main component of parasitic well loss.
- Simply enlarging the diameter of the wellbore might be the most cost-effective method of minimizing well loss.

ACKNOWLEDGMENTS

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REFERENCES

Atkinson, L.C., 1985, A Laboratory and Numerical Investigation of Steady-State, Two-Regime, Radial Flow to a Well from Rough, Horizontal, Deformable Fractures, Ph.D. thesis, Memorial University of Newfoundland, in preparation.

Baker, W.J., 1955, Flow in Fissured Formations, Proceedings of 4th World Petroleum Congress (Rome), Sec II/E, Paper 7, pp. 379-393.

Brereton, N.R., 1979, Step-Drawdown Pumping Tests for the Determination of Aquifer and Borehole Characteristics, Water Research Centre (England) Technical Report TR103, 26 pp.

Caswell, B., 1985, Better Yields from Large Diameter Wells in Crystalline Rock, Water Well Journal, Vol. 39, No. 4, pp. 34-36.

Da Cruz, P.T., and De Quadros, E.F., 1984, Analysis of Water Losses in Basaltic Rock Units, Proceedings of 5th Congress of International Society for Rock Mechanics (Melbourne), pp. B119-B123.

Gale, J.E., Rouleau, A., and Atkinson, L.C., 1985, Hydraulic Properties of Fractures, Hydrologeology of Rocks of Low Permeability: Proceedings of 17th International Congress of International Association of Hydrogeologists (Tucson), 17 pp.

Gale, J.E., 1982, Assessing the Permeability Characteristics of Fractures, Recent Trends in Hydrogeology: Geol. Soc. America Special Paper No. 189, pp. 163-181.

Lomize, G.M., 1951, Water Flow in Jointed Rock, Gosenergoizdat (in Russian; translation of Ch. 7, "Flow of Water in Fractures of Variable Cross Section and Curvilinear Outline", obtained courtesy of Lawrence Berkeley Laboratory), pp. 84-92.

Louis, C., 1969, A Study of Groundwater Flow in Jointed Rock and its Influence on the Stability of Rock Masses, Rock Mechanics Research Report No. 10., Imperial College, 90 pp.

Mackie, C.D., 1982, Multi-Rate Testing in Fractured Formations, Papers of the Groundwater in Fractured Rock Conference (Canberra), pp. 139-150.

Murphy, H., and Pearce, R., 1980, Pressure Losses at the Transition from Radial Flow within Fractures to Longitudinal Flow within Production Wells, Los Alamos Scientific Laboratory Report LA-8197-MS, 22 pp.

Norris, S.E., 1976, Change in Drawdown Caused by Enlarging a Well in a Dolomite Aquifer, Ground Water, Vol. 14, No. 4, pp. 191-193.

Pearce, R., and Murphy, H., 1979, Roughness Effects on Flow in Hydrualic Fractures - Survey of the Literature, unpublished report, Los Alamos Scientific Laboratory, 32 pp.

Rissler, P., 1978, Determination of the Water Permeability of Jointed Rock, Volume 5: Publications of the Institute for Foundation Engineering, Soil Mechanics, Rock Mechanics, and Water Ways Construction, Rheinisch-Westfalien Technische Hochschule, Aachen (West Germany), 150 pp.

Uhl, V.W., Jr., Joshi, V.G., Alpheus, A., and Sharma, G.K., 1976, The Application of Step-Drawdown Pumping Tests to Water Wells in Consolidated Rock Aquifers, Indian Geohydrology, Vol. 11, Nos. 3-4, pp. 124-149.

Williamson, W.H., 1982, The Use of Hydraulic Techniques to Improve the Yield of Bores in Fractured Rocks, Papers of the Groundwater in Fractured Rock Conference, (Canberra), pp. 273-282.

Williamsom, W.H., and Woolley, D.R., 1980, Hydraulic Fracturing to Improve the Yield of Bores in Fractured Rock, Australian Water Resources Council Tech. Paper No. 55, 73 pp.