SUBMODELS OF GROUNDWATER-SURFACE WATER INTERACTION FOR THE ANALYSIS OF REGIONAL WATER POLICIES IN OPEN-PIT LIGNITE MINING AREAS

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ABSTRACT

The paper outlines the approach to groundwater-surface water submodels for the analysis of regional water policies in a test region in the German Democratic Republic.

In open-pit lignite mining areas processes of infiltration and exfiltration between ground- and surface water resources are frequently an important factor for the regional water balance. For example, infiltration losses of surface water caused by mine dewatering reduce the water supply for downstream water users and increase the groundwater pumpage for dewatering.

For the analysis of regional water policies in lignite mining areas such processes have to be considered carefully. The submodels to be developed have to reflect the natural processes sufficiently accurately but should be as simple as possible.

An introductory methodological background is given and ways for model reduction based on computations with comprehensive flow models are discussed. The proposed methodological approaches are demonstrated for the development of a submodel for the management of remaining pits in lignite mining areas.

METHODOLOGICAL BACKGROUND

Discussions in the introductory paper by Kaden et al. (1985b) emphasize the apparent need for the development of reduced models of regional flow processes. The demand for such models results from the concept of complex Decision Support Model Systems (DSMS). Furthermore, the need for reduced models may be interpreted as a modified form of the requirement to apply ***problem-adequate*** models. The integration of a comprehensive regional model as an element into a DSMS is not only a computer problem, but primary a question of reasonable means.

Generally, there are two different approaches for model reduction:

(1) Simplification of the mathematical model of the flow process in such a manner that the simplified model can be used as an element of the DSMS.

(2) Utilization of the comprehensive model as an analogue of the natural processes for synthetic data generation. Reduced models are derived by fitting to these synthetic data.

Both procedures have advantages and disadvantages. Obviously, the type of the required submodel and the way for model reduction depend on the model level under consideration.

The ***first approach*** results in general applicable solutions supposed it does not include empirical object-specific data. The latter is the case for typical hydrological applications. For the model development especially in the operational hydrology the completeness and reliability of observed time series become fundamental.

The ***second approach*** is directed towards a rational storage and processing of computational results of the comprehensive regional model. It is aspired to realize a similar accuracy as with the comprehensive model. This is simple if the results of the model can be processed as a time series. Such an approach is reasonable above all for time steps $\Delta t \geq 1$ *year*. The state of the hydrological system is described by discrete time functions (state descriptive functions), in many cases depending on only a few parameters. Typical examples of such submodels are given by Peukert et al. (1985).

Commonly, the reduced model has to be not only an effective means for storage of computed data, but it has to simulate the functional relationships for defined parts of the comprehensive model. With respect to the interannual water balance situation it is necessary for monthly and smaller time-steps to consider all groundwater/surface water interaction processes as systems variables. Therefore, adequate submodels involving the history of the process are needed. In most cases for that a linearization becomes necessary.

Only box-models may be used as reduced models in this sense, conceptual or black-box type. With regard to the transition function box-models may be deterministic or stochastic. In the field of groundwater management deterministic box-models are dominant.

Another point of view is the way for obtaining the transition function. In the case of conceptual box-models the transition function is derived from special analytical solutions of the systems-descriptive model. Therefore, the parameters of this type of models allow for a clear physical interpretation. Such models have the advantage that they may be derived for regions even if no comprehensive model is available.

A physical interpretation is not possible and not necessary for black-box models. The parameters of transition functions are obtained by fitting empirical or theoretical formulas to observation data or calculations using the comprehensive model. With respect to the compatibility of models with different time steps, however, it is helpful, if the coefficients of the black-box model are given as explicite functions of the time step under consideration. Grey-box models are a compromise between conceptual box-models and black-box models.

In Figure 1 an overview of different ways for model reduction is given. As a typical example in the following the simulation of the management of a remaining pit is discussed.

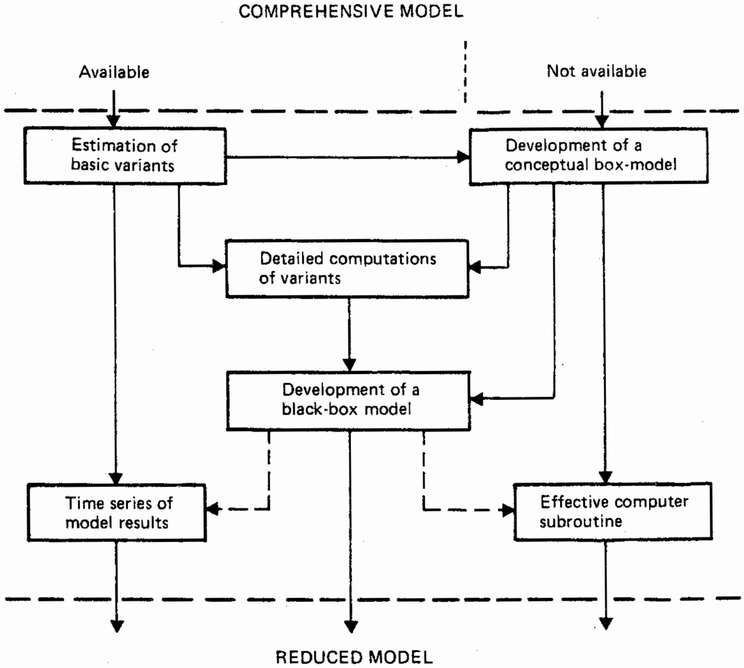

Figure 1: Ways for model reduction

REMAINING PIT MANAGEMENT

Analysis of the Problem

The hydrological utilization of the remaining pits in mining areas is the most preferable solution for a reasonable recultivation of the mining areas, to avoid water deficits, and to satisfy flood protection. Two major stages have to be distinguished, the stage of recharging the remaining pit, and the management stage for its utilization in water management.

Recharge stage

The management of remaining pits takes place within a "usable storage layer". In order to get the water table of the remaining pit within this layer it is necessary to recharge the remaining pit after abandoning the drainage wells around the open-pit mine. This can either be done by natural groundwater inflow or additionally by artificial surface water or mine water inflow. The latter results in water losses by infiltration from the remaining pit into the aquifer.

Management stage

After reaching the usable storage layer the remaining pit can be used as a water reservoir. In such a case, the management is analogously to reservoir management. One difference is that the storage basin is located in the by-pass of the stream. Due to this, a pumping station can be included in the management to transfer water between the remaining pit and the stream especially for flow augmentation.

Detailed computations of variants of the recharge and management stage have been done with the comprehensive groundwater flow model described by Peukert et al. (1985) in order to get comprehensive synthetical data about the nonlinear process of remaining pit management as a base for model reduction. For results see Figures 3 and 6.

The reduced models have to be designed to describe the dynamic behavior of the water table in the remaining pit in both the recharge and management stage.

Grey-Box Model

The model structure is derived by the help of a block concept subdividing the area under consideration into three blocks:

- the remaining pit,
- the aquifer around the remaining pit (GWL1) which is directly influenced by the remaining pit,
- the neighbored part of the aquifer (GWL2) which is undisturbed by the other blocks (see Figure 2).

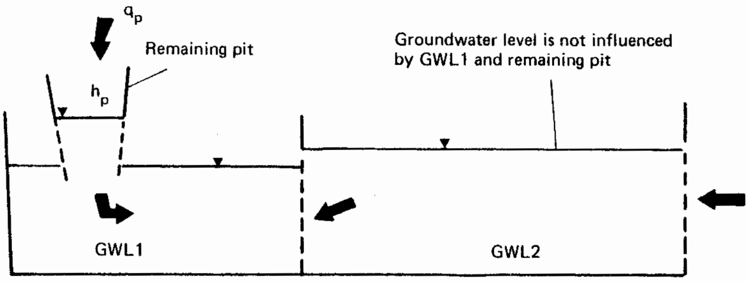

Figure 2: Block structure of the grey-box model

The water table in the remaining pit and the groundwater table are the state variables, assumed to be constant within their blocks. The blocks are connected by the continuity equation and a kinetic equation. In order to get a simple model structure two assumptions have been formulated which enable an approximate linearization of the problem:

(1) The horizontal area of the remaining pit at any water level is proportional to the corresponding area of exchange between the remaining pit and the GWL1. Such an assumption permits the linearization of the dynamic behavior if the reaction of the storage GWL1 on the remaining pit is negligible.

(2) The influence of the remaining pit management on the dynamic systems behavior is small and is assumed to be approximately linear. For example this holds true if the variation of the exchange area of the remaining pit is small in relation to its water table.

Especially within the usable storage layer (from $108\,m < h_p < 118\,m$ in the case of the GDR Test Area) both assumptions are justified. Possible influences of external boundary conditions on the dynamic systems behavior are separated by subtraction of two different management variants with the same external boundary conditions.

Based on the assumptions above we obtain for the dynamic behaviour of the water table for two interacting storages (Kindler 1972):

$$D_1 \cdot D_2 \cdot \frac{d^2\tilde{h}_p}{dt^2} + (D_1 + D_2) \cdot \frac{d\tilde{h}_p}{dt} + \tilde{h}_p = K \cdot q_p \tag{1}$$

with D_1, D_2 - time constants
K - proportionality constant
q_p - constant inflow into (> 0) or discharge from ($<= 0$) the remaining pit
$\tilde{h}_p$ - difference between the actual water table and that for natural recharge.

The homogeneous solution of the differential equation (1) can be given as a homogeneous recurrence equation of second order:

$$\tilde{h}_p(t_j) = (P_1 + P_2) \cdot \tilde{h}_p(t_{j-1}) - P_1 \cdot P_2 \cdot \tilde{h}_p(t_{j-2}) \tag{2}$$

$$\text{with:} \quad P_1 = e^{\left(-\frac{\Delta t}{D_1}\right)}, \; P_2 = e^{\left(-\frac{\Delta t}{D_2}\right)}, \; \Delta t = t_j - t_{j-1}$$

After integration of the differential equation (1) with the initial conditions

$$\tilde{h}_p(0) = 0 \; , \quad \frac{d\tilde{h}_p(0)}{dt} = \frac{\Delta t}{D_1 \cdot D_2} \cdot K \cdot q_p \tag{3}$$

we get the transition function $S(\Delta t)$ in the following form:

$$S(\Delta t) = 1 - c_1 \cdot P_1 - c_2 \cdot P_2 \; , \text{ with } c_1 = \frac{\Delta t - D_1}{D_2 - D_1} \; , \; c_2 = 1 - c_1 \tag{4}$$

Through superposition of the discrete time-dependent inflow into or discharge from the remaining pit we obtain based on the homogeneous solution:

$$h_p(i) = h_p^0(i) + (P_1 + P_2) \cdot \tilde{h}_p(i-1) - P_1 \cdot P_2 \cdot \tilde{h}_p(i-2) + \\ + (1 - c_1 \cdot P_1 - c_2 \cdot P_2) \cdot K \cdot q_p(i) + (P_1 \cdot P_2 - c_2 \cdot P_1 - c_1 \cdot P_2) \cdot K \cdot q_p(i-1) \tag{5}$$

with i - time step in years
$h_p^0(i)$ - water table in the remaining pit for natural recharge at the end of the year i
$h_p(i)$ - water table in the remaining pit at the end of the year i
$q_p(i)$ - inflow into or discharge from the remaining pit within the year i in $\frac{Mill.\, m^3}{year}$

Eq.(5) has three parameters - the two time constants D_1, D_2 and the proportionality constant K. These parameters are quantified by adaptation of Eq. (5) to discrete annual values of the water level in the remaining pit (calculated by means of the comprehensive grundwater flow model) for different management variants.

To apply the Eq.(5) also for water tables below the usable storage layer it was empirically modified with the arbitrary function $\gamma(i)$.

$$\gamma(i)=\begin{cases}2.22-0.004\cdot q_p(i) & \text{for } i\le ip+1\\ 1.45-0.003\cdot q_p(i) & \text{for } i=ip+2\\ 1 & \text{for } i>ip+2\end{cases} \tag{6}$$

with ip - year of opening the remaining pit.

Based on that the *grey-box model* for yearly time steps gets the following form:

Annual model

$$h_p(i)=h_p^0(i)+a_1\cdot\tilde{h}_p(i-1)+a_2\cdot\tilde{h}_p(i-2)+ \tag{7}$$
$$+b_0\cdot\gamma(i)\cdot q_p(i)+b_1\cdot\gamma(i-1)\cdot q_p(i-1)$$

with $\tilde{h}_p(i)=h_p(i)-h_p^0(i)$.

Through modification of the time interval in the annual model (modification of parameters P_1 and P_2 in Eq.(2)) we get the following monthly model:

Monthly model

$$h_p(i,k)=h_p^0(i,k)+a_1\cdot\tilde{h}_p(i,k-1)+a_2\cdot\tilde{h}_p(i,k-2)+ \tag{8}$$
$$+b_0\cdot\gamma(i)\cdot q_p(i,k)+b_1\cdot\gamma(i-1)\cdot q_p(i,k-1)$$

$$\tilde{h}_p(i,k)=h_p(i,k)-h_p^0(i,k)$$
$$h_p^0(i,k)=h_p^0(i-1,12)+(h_p^0(i,12)-h_p^0(i-1,12))\cdot\frac{k}{12} \tag{9}$$

with k - number of month, $k=1,\ldots,12$
$h_p(i,k)$ - water level in the remaining pit at the end of month k in the year i in meters
$q_p(i,k)$ - constant inflow into or discharge from the remaining pit within the month k in the year i in $\frac{Mill.\ m^3}{year}$.

In Figure 3 the water table in the remaining pit is depicted for different management variants comparing the results of the reduced model and of the comprehensive groundwater flow model.

Conceptual Box-Model

In simplifying its geometry, a remaining pit can be considered as a well with a large diameter. Consequently it is possible to use analytical solutions of the well hydraulics as a transition function. The inner boundary condition of the classical THEISS-solution ($r\to 0$) has to be replaced by an adequately modified condition because the storage effect of the "well" (remaining pit) is not negligible (see Figure 4).
According to Cooper et al. (1967) we get the following approach applying the **Laplace**-transformation:

Differential equation:

$$\frac{\delta^2 h}{\delta r^2}+\frac{1}{r}\cdot\frac{\delta h}{\delta r}=\frac{S}{T}\cdot\frac{\delta h}{\delta t} \tag{10}$$

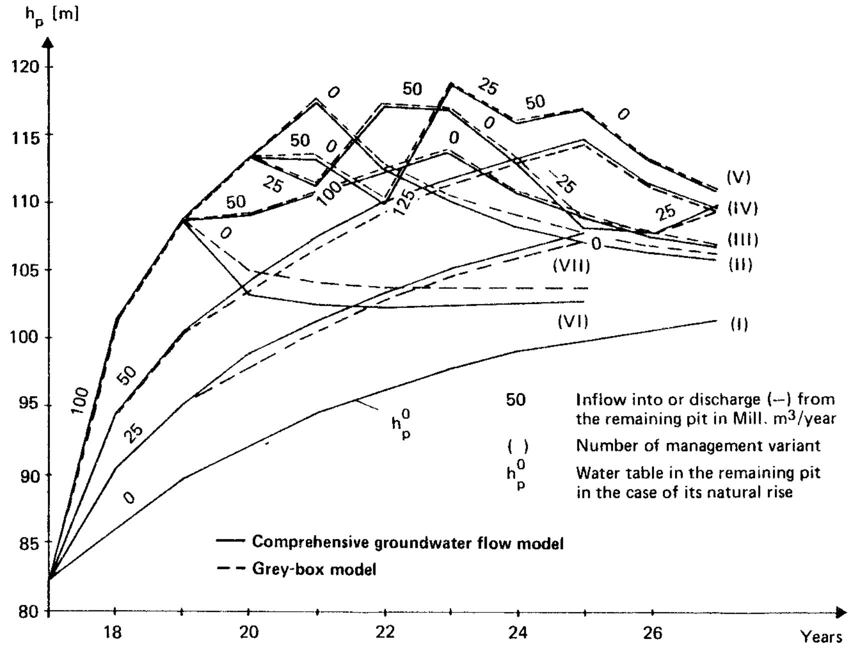

Figure 3: Water table in the remaining pit for different management variants - grey-box model

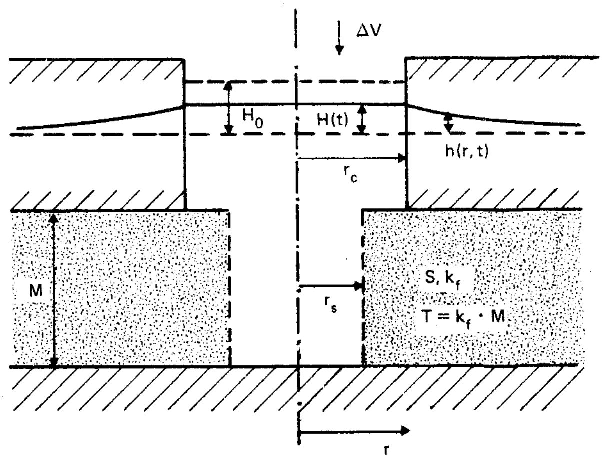

Figure 4: Idealized representation of a finite-diameter well

Boundary and inital conditions:

$$h(r_s,t) = H(t) \quad , \quad h(\infty,t) = 0 \quad , \quad h(r,0) = 0 \tag{11}$$

$$H(0) = \frac{\Delta V}{\pi \cdot r_c^2} \tag{12}$$

$$2\pi r_s T \cdot \frac{\delta h(r_s,t)}{\delta r} = \pi r_c^2 \cdot \frac{\delta H(t)}{\delta t} \tag{13}$$

Solution of **Laplace**-*transformed differential equation*:

$$\bar{h}(r,p) = \frac{S \cdot r_s \cdot H(0) \cdot K_0(\sigma \cdot r_s)}{\sigma \cdot T \cdot [\sigma \cdot r_s \cdot K_0(\sigma \cdot r_s) + 2\alpha \cdot K_1(\sigma \cdot r_s)]} \tag{14}$$

with K_0, K_1 – **Bessel**–functions
r – space coordinate in m
t – time in sec.
r_c, r_s – see Figure 4
α – geohydraulic time constant in sec/m^2
S – storage coefficient
T – Transmissivity in m^2/sec

and $$\alpha = \frac{r_s^2}{r_c^2} \cdot S \qquad \sigma = \sqrt{p \cdot \frac{S}{T}} = \sqrt{p \cdot \alpha} \quad . \tag{15}$$

Solution of the problem by inverse **Laplace**-*transformation*:

$$H(t) = h(r_s,t) = F \cdot H(0) = L^{-1}\left\{\bar{h}(r_s,p)\right\} \cdot H(0) \tag{16}$$

The factor F results from the inverse **Laplace**-transformation. From the analytical inverse transformation we get according to Carslaw, Jaeger (1959):

$$F = 8\alpha / \pi^2 \int_0^\infty \frac{e^{-\beta u^2/\alpha}}{u \cdot \Delta(u)} \cdot du \tag{17}$$

with

$$\beta = \frac{T \cdot t}{r_c^2} \quad , \quad \alpha = \frac{r_s^2}{r_c^2} \cdot S \tag{18}$$

$$\Delta(u) = [u \cdot J_0(u) - 2\alpha \cdot J_1(u)]^2 + [u \cdot Y_0(u) - 2\alpha \cdot Y_1(u)]^2 \tag{19}$$

and J_0, J_1, Y_0, Y_1 - generalized **Bessel**-functions.

For management modeling of the remaining pit the solution Eq.(16) is in the given form not applicable because a few typical conditions have not been considered:

(1) Variations of groundwater dynamics due to external boundary conditions;

The influence of external boundary conditions is eliminated by the help of separation calculations. The actual variation of the storage volume v_p of the remaining pit results on the one hand from the inflow/outflow due to external boundary conditions (natural recharge q_p^0) and on the other hand from intakes/discharges Δq_p

and exfiltrations/infiltrations qi_p resulting therefrom (see Figure 5). The following balance equation holds for a planning horizon from time t_B to t_E:

$$v_p(t_E) = \int_{t_B}^{t_E} (q_p^0 + \Delta q_p - qi_p)\, dt + v_p(t_B) \tag{20}$$

with $\quad h_p(t_p) = fh_p(v_p(t_p))\,. \quad$ (21)

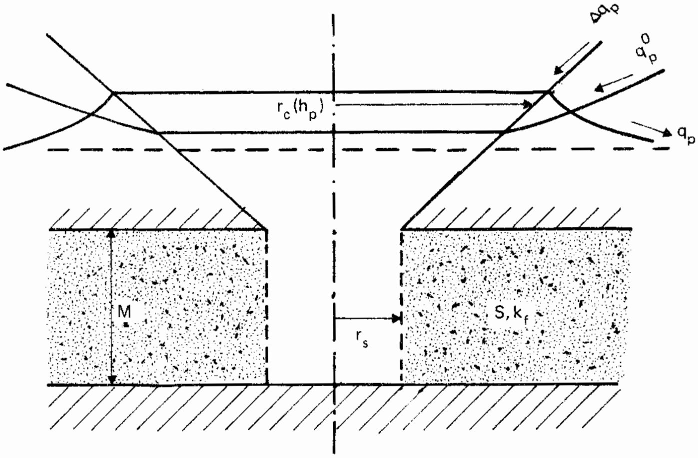

Figure 5: Separation of balance components

(2) Differing geometry of the remaining pit from the cylindrical well form (nonlinear dependency between storage water table and volume);

The geometrical deviation of the remaining pit from a cylindrical form is characterized by the relationship $r_c = f(h_p)$. This nonlinearity is eliminated updating the radius r_c by step (step by step linearization).

(3) Unconfined flow conditions;

The unconfined flow condition (transmissivity $T = f(h_p)$) is simplified introducing a mean constant transmissivity.

(4) Time-variable management (artificial inflow);

The consideration of the time-variable management is possible on the basis of the superposition principle by the use of the convolution operation.

(5) Consideration of an additional hydraulic resistance reflecting the transformation of flow from vertical to horizontal direction;

Within the comprehensive groundwater flow model the remaining pit is modeled in the form of an inner boundary condition of third kind. In order to consider this in the box-model, the relevant radius for the exchange area, $r_{s,old}$, is reduced:

$$r_{s,new} = \frac{r_{s,old}}{e^{2\pi T R_{hydr}}} \tag{22}$$

The reciprocal value of R_{hydr} results as a sum of the reciprocal additional hydraulic resistances (parallel circuit) used in the boundary condition in the comprehensive flow model.

Based on all mentioned modifications we obtain the following time-discrete algorithm with $t_k = k \cdot \Delta t$, $k_B \leq k \leq k_E$:

$$v_p(t_E) = v_p(t_B) + \sum_{k=k_B}^{k_E-1} \left| v_p^0(t_k) + \Delta v_p(t_k) - vi_p(t_k) \right| \tag{23}$$

The individual components are determined as follows:

$$v_p^0(t_k) = fv_p(h_p^0(t_k+1)) - fv_p(h_p^0(t_k)) \tag{24}$$

$$\Delta v_p(t_k) = \Delta q_p(t_{k+\frac{1}{2}}) \cdot \Delta t \quad , \quad vi_p(t_k) = qi(t_k) \cdot \Delta t \tag{25}$$

with

$$qi(t_k) = \sum_{l=k_B}^{k} fv_p(h_p(t_l) + \Delta h_p(t_l)) - fv_p(h_p(t_l) + F(t_{k+1}-t_l) \cdot \Delta h_p(t_l)) \tag{26}$$

$$F(t_k - t_l) = L^{-1} \left\{ \frac{S \cdot r_s \cdot K_0(\sigma r_s)}{\sigma \cdot T \cdot [\sigma \cdot r_s \cdot K_0(\sigma r_s) + 2 \frac{r_s^2}{r_{c,l}^2} K_1(\sigma r_s)]} \right\} \tag{27}$$

$$r_{c,l} = \sqrt{\frac{\Delta v_p(t_l)}{\pi \cdot \Delta h_p(t_l)}} \tag{28}$$

$$\Delta h_p(t_l) = fh_p(v_p(t_l) + \Delta v_p(t_l)) - h_p(t_l) \tag{29}$$

h_p^0 is the water level in the remaining pit for natural recharge, σ see Eq.(15). The inverse **Laplace**-transformation is done numerically. On the basis of the numerical integration the computing time is reduced by a factor of about 15 as compared with the computation of F in Eq. (17).

The developed program has been tested for the remaining pit management in the GDR Test Area for monthly time steps. In Figure 6 the computational results for different management variants are compared with those of the comprehensive model.

Because the response of the conceptional block-model agrees well with that of the comprehensive flow model for monthly constant management, we conclude from these results at the reliability of the conceptional model in the case of monthly varying management. Figure 7 shows the comparison between computing results executed in different ways for monthly management.

CONCLUSIONS

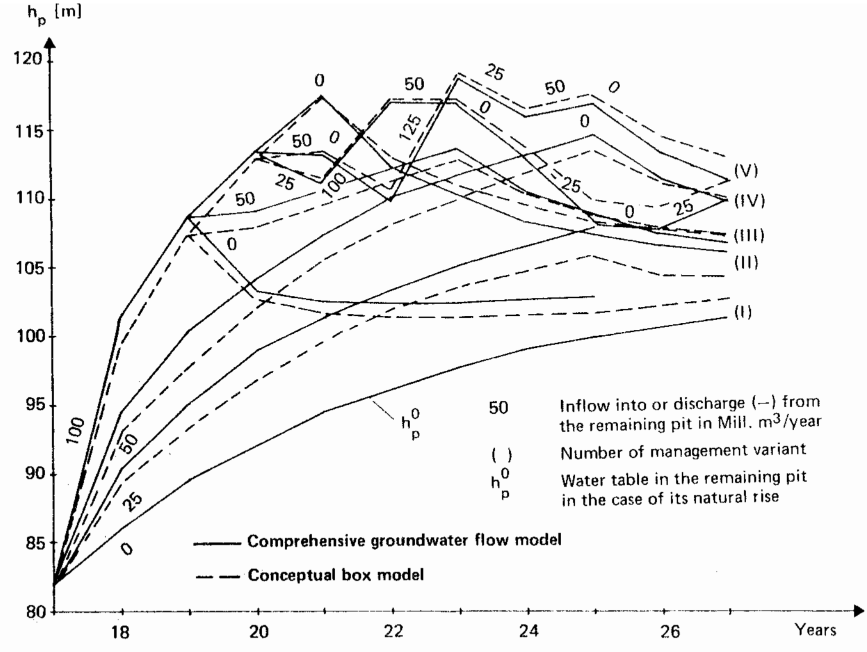

Figure 6: Water table in the remaining pit for different management variants - conceptual box-model

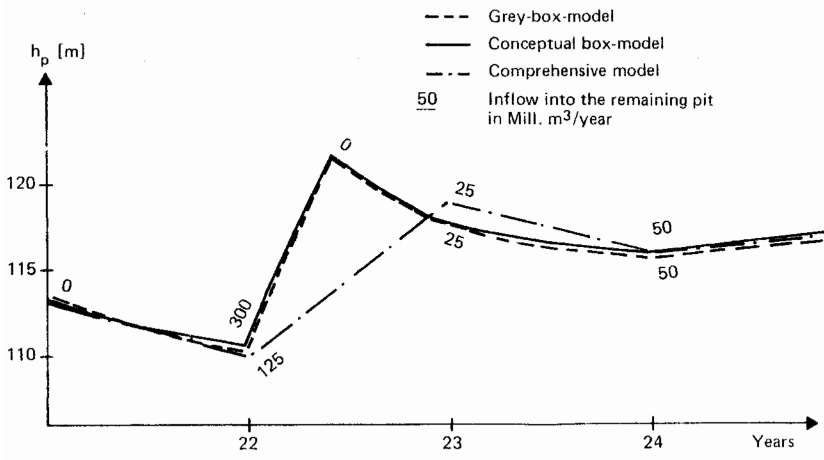

Figure 7: Comparison of model results

The presented examples demonstrate the applicability of the developed approaches. In similar form further interaction processes between groundwater and surface water have been studied:

- Impact of remaining pit management on mine drainage (grey-box model)
- Exchange processes between a stream and groundwater in lignite mining areas (conceptual box-model).

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