Probabilistic Estimation of Aquifer Properties and Non-Linear Well Losses to Support Dewatering Forecasts and Planning

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Abstract

We review and apply the methods used by Benson *et al.* (2026) who use a Bayesian Markov-chain Monte Carlo (MCMC) framework to estimate aquifer properties and non-linear well losses from pumping. Results show strong parameter correlation and non-Gaussian uncertainty, which are not captured by conventional deterministic methods. A synthetic dewatering example links parameter uncertainty to operating costs across a range of pumping rates. A volume-price risk analysis directly links pumping targets to the probability of meeting budget constraints, enabling rapid and transparent evaluation of production-cost trade-offs. The approach is implemented in Python and provides a fast, low-cost tool for risk-informed planning.

Keywords: MCMC, pump-test, uncertainty, well efficiency

Introduction

Reliable estimation of aquifer and well performance parameters is critical for the design and planning of mine dewatering systems. Conventional pumping test interpretation typically relies on deterministic calibration, producing single-value estimates of transmissivity, storativity (or specific yield), and well-loss coefficients. These parameters are often highly correlated and non-unique, which can lead to overconfident predictions of drawdown, well efficiency, and associated pumping costs. In mine dewatering applications, where pumping may be sustained over the life of a project, pumping energy can represent a substantial fraction of total operating expenditure, making parameter uncertainty directly consequential to project economics. Where aquifer parameters are poorly constrained, uncertainty in drawdown and well efficiency predictions translate directly into uncertainty in energy consumption and operation costs. This financial sensitivity is greatest during feasibility and pre-feasibility studies, where design decisions are made under high uncertainty. A probabilistic framework for pump test interpretation that quantifies uncertainty in pumping costs as a function of production rate allows mine planners to better understand the potential range in dewatering costs. This is particularly valuable when: (1) electricity cost represents a large fraction of total mine operating expenditure; (2) aquifer parameters are poorly constrained due to limited pumping test data, resulting in a wide range of plausible drawdown and cost outcomes; or (3) a go/no-go planning decision depends on whether a target pumping rate can be sustained within acceptable cost bounds.

Benson *et al.* (2026) present a Bayesian Markov-chain Monte Carlo (MCMC) framework for estimating aquifer parameters and non-linear well-loss coefficients from variable-rate pumping tests. Their results show that parameter estimates are frequently non-Gaussian and strongly correlated, particularly between transmissivity and storativity, and between linear and non-linear well-loss parameters. This correlation reflects the nonlinear relationship between parameters and the drawdown response and gives rise to non-uniqueness: multiple distinct parameter combinations can reproduce the observed data equally well. The framework links parameter uncertainty to uncertainty in well efficiency (the ratio of theoretical to actual drawdown at a given pumping rate) and pumping costs.

Methods

Aquifer Response and Non-Linear Well Losses

The model is based on the Theis (1935) solution for radial flow to a well:

$$s(t) = \frac{Q}{4\pi T} W(u),$$

where drawdown $s(t)$ depends linearly on pumping rate Q, transmissivity T, and the well function $W(u)$, that depends on $u = r^2 S/(4Tt)$, where r is the radius of measurement and S is aquifer storativity (Kruseman and Ridder 2000). For variable-rate pumping, the linear response is expressed using convolution (denoted by *):

$$s(t) = B(t) \star Q(t) \text{ where } B(t) = \frac{1}{4\pi T} \frac{d}{dt} W(u).$$

Step-drawdown tests commonly exhibit additional drawdown that increases non-linearly with pumping rate due to turbulent flow near the well. This is represented as: $s(t) = B(t)*Q(t) + CQ^p(t)$ where C is a non-linear well-loss coefficient and p is typically greater than 2 (Cooper and Jacob 1946; Rorabaugh 1953; Benson 2024). This equation generalises the classical quadratic well-loss model and better captures observed behaviour in step tests.

Benson *et al.* (2026) introduce a discharge correction to account for wellbore storage effects, improving early-time model fit and parameter estimation. The corrected discharge is:

$$Q_c = sign(Q)(Q - \pi r_w^2 \frac{ds}{dt})$$

where r_w is pumping well radius and the drawdown rate is best estimated by backward finite differences of the measured data. Drawdown is now:

$$s(t) = B(t) \star Q(t) + sign(Q) C Q_c^p(t).$$

Bayesian Inference and MCMC Sampling

Bayesian inference is applied using MCMC sampling to estimate posterior distributions of model parameters. The Bayesian framework updates prior parameter distributions using observed data via Bayes' theorem:

$$p(M|D) \propto p(D|M)p(M)$$

where $p(M|D)$ is the posterior probability density function (PDF) of model M given the data D, $p(D|M)$ is the likelihood that the data D came from model M and $p(M)$ is the prior PDF of model M. Sampling is performed with an ensemble sampler from emcee (Foreman-Mackey *et al.* 2013, https://emcee.readthedocs.io/en/stable/). This sampler was designed to handle correlated and non-Gaussian parameter values with multiple walkers simultaneously. Convergence is assessed using chain length relative to autocorrelation time and diagnostics such as the Gelman-Rubin statistic (Gelman and Rubin 1992).

Cost Estimation and risk analysis

Hydraulic uncertainty is linked to operational costs through energy requirements for pumping. Power demand is calculated from total dynamic head (static lift plus losses), following methodology in the State of Colorado Water Plan (Starosta *et al.* 2019). Parameter sets drawn from the posterior distribution are used to estimate annual operating costs for standardized pumping scenarios.

Synthetic Mine Dewatering Example

The framework is applied to a synthetic high-capacity dewatering well. Posterior distributions of S, T, C, and p are used to simulate drawdown across a range of pumping rates, which are converted to annual production volumes. These are used to estimate energy consumption and operating costs with uncertainty bounds. A volume–price risk analysis is conducted to quantify the probability of operating below specified cost thresholds and the risk of exceeding them, providing a decision-support framework for selecting pumping strategies under uncertainty.

Results

Benson *et al.* (2026) show that aquifer and well-loss parameters may exhibit strong correlation and non-uniqueness. The posterior distributions for a typical pumping well (Figure 1) demonstrate:

- Negative correlation between transmissivity and storativity;
- Strong negative correlation between non-linear well-loss parameters (C and p);
- Non-gaussian, elongated (“banana-shaped”) posterior parameter distributions;

These results indicate that multiple parameter combinations can produce observed data equally well, limiting the reliability of single best-fit solutions.

Accounting for wellbore storage improves early-time model fit and reduces bias by preventing wellbore storage effects from being incorrectly attributed to non-linear well losses. This leads to more realistic distributions of well efficiency and operating cost. Bayesian assessment of well efficiency

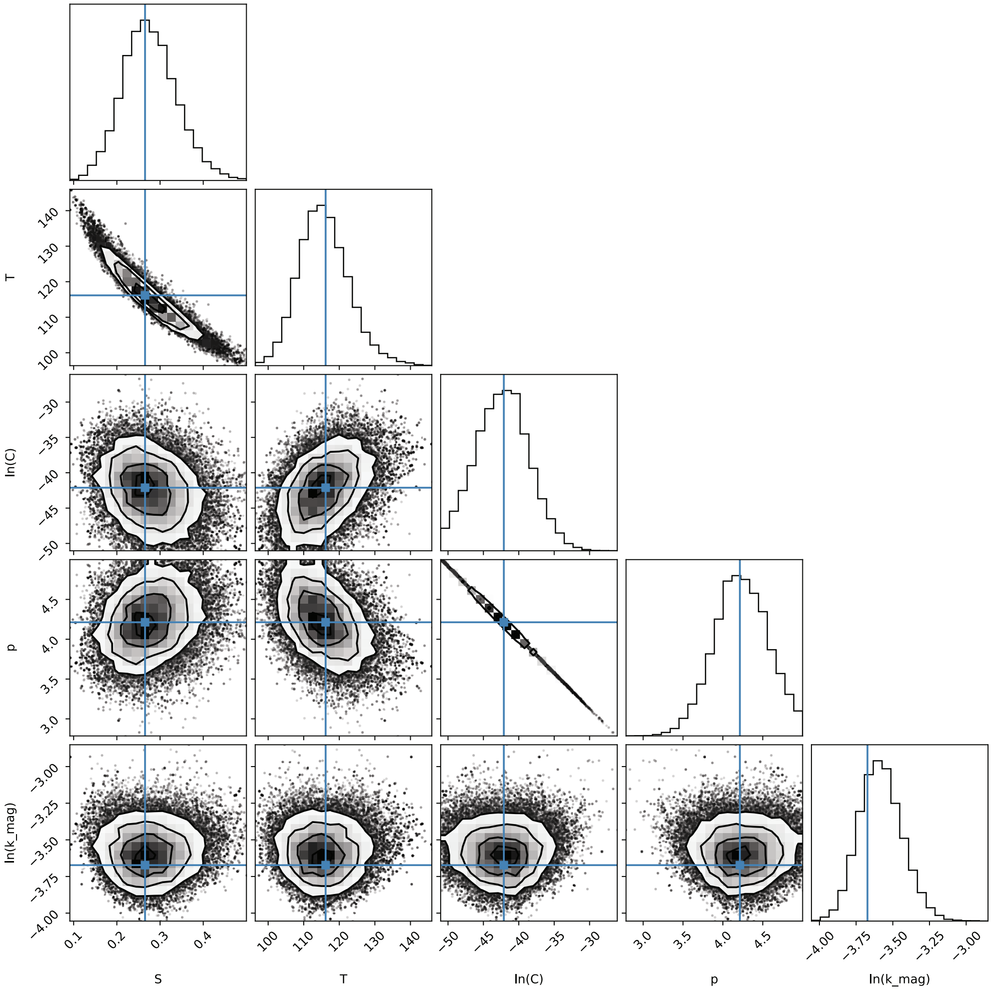

Figure 1 Corner plot of 1-D and 2-D marginal densities of each parameter with all others for well CR-226. "Best-fit" values from Marquardt-Levenberg optimization are shown in blue. Taken from Benson et al. *(2026).*

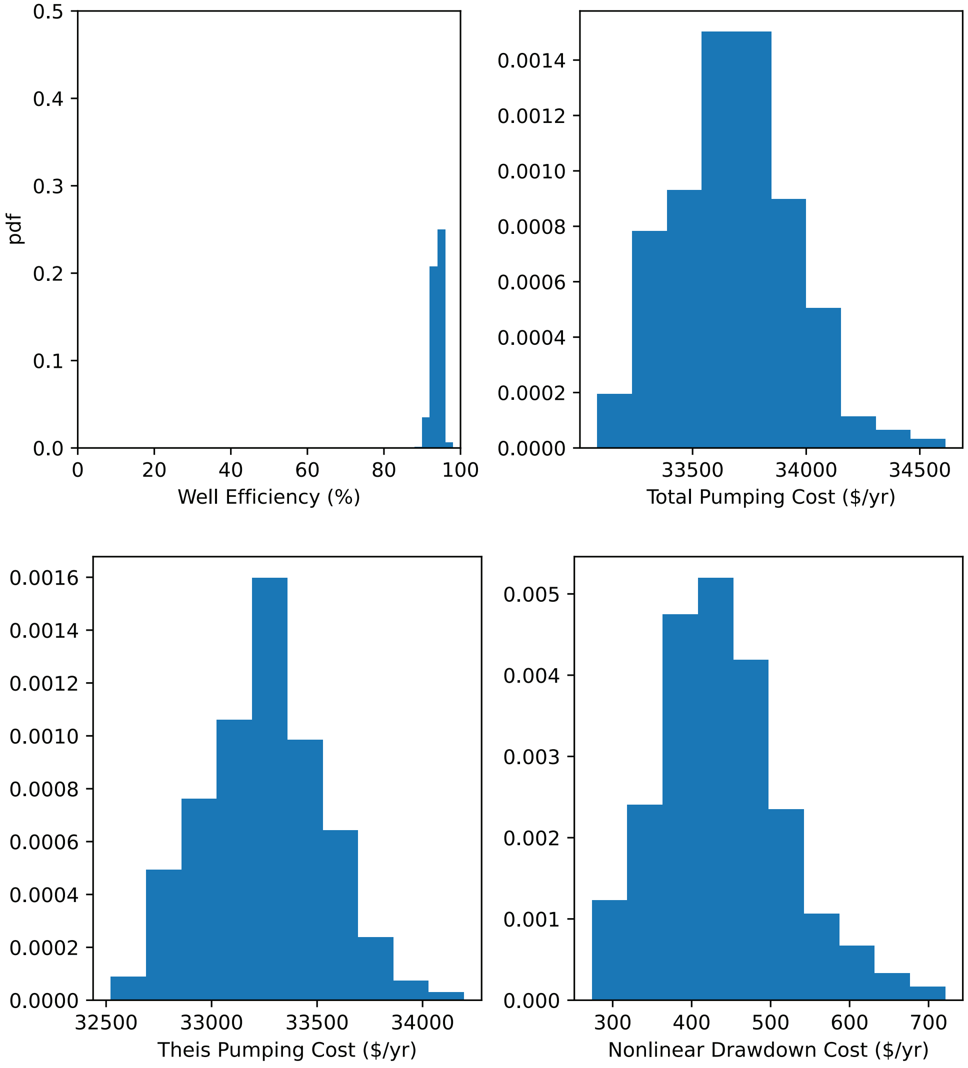

***Figure 2** Example histograms of forward runs taken from the Markov chain for well CR-226. Taken from Benson* et al. *(2026).*

and associated cost distributions can be seen in Figure 2 (Benson *et al.* 2026).

Application to Mine Dewatering: Synthetic Evaluation

A synthetic dewatering well was analysed with representative characteristics (e.g., 300 m depth, 73 m screen interval, 0.15 m radius). Posterior parameter distributions derived from the well CR 226 step-drawdown test but revised well design parameters were used to simulate system performance. Results (Figure 3) show that both energy use and cost increase non-linearly with pumping rate due to:

- Increased drawdown at higher extraction rates;
- Non-linear well losses;
- Frictional head losses;

Cost uncertainty also increases with production rate, leading to a widening spread of possible outcomes.

The volume-price risk analysis (Figure 4) shows the probability of maintaining a

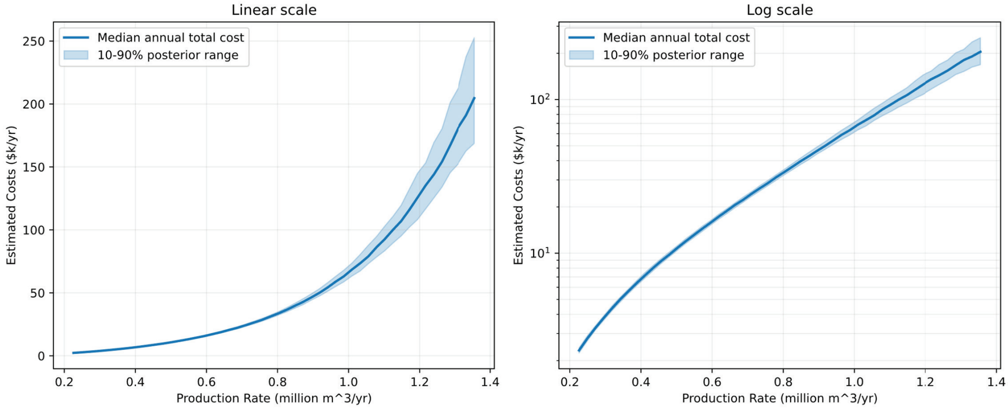

Figure 3 *Estimated costs for a range in production rates under uncertainty.*

cost threshold for different annual pumping volumes. For example, at a threshold of approximately $105.8k/year, the probability of meeting the threshold for different annual pumping targets is:

- At 0.89 million m^3/year: ~100% probability of meeting the threshold;
- At 1.12 million m^3/year: ~74% probability of success;
- At 1.24 million m^3/year: ~0.25% probability of success;

The analysis shows a rapid change in the probability of meeting a cost threshold over a relatively narrow range of pumping rates. Note, this example assumes that the aquifer and pumping system can sustain the required discharge rates.

Discussion

These results highlight the limitations of deterministic pumping test interpretation for dewatering design, where parameter correlation and non-uniqueness can lead to underestimation of uncertainty in system performance. Conventional pumping test analyses produce a single best-fit parameter set and therefore a single predicted outcome for each pumping rate, with no indication of the confidence that should be placed in those estimates. In contrast, the Bayesian MCMC framework captures the full range of plausible parameter combinations and translates this uncertainty into predictions of drawdown, energy use, and cost. For the synthetic example presented here, the

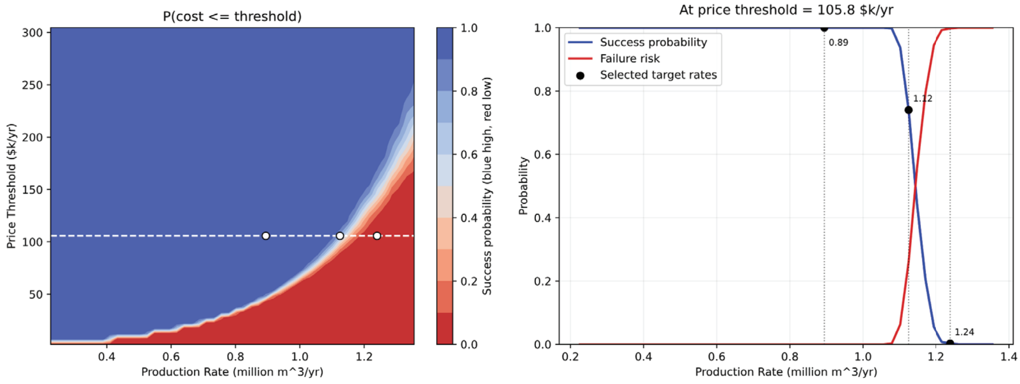

Figure 4 *Estimated costs for a range in production rates under uncertainty.*

deterministic approach would suggest a single annual cost at each pumping rate, for example, $85,700 at a pumping rate of 1.08 Mm3/year, whereas the MCMC framework reveals that the true cost could range from approximately $70,000 to $102,000 at that same rate, representing a 37% spread around the deterministic estimate (see Figure 3). This range reflects the combined influence of parameter correlation and non-uniqueness, neither of which is captured by a single best-fit solution. The synthetic example shows that cost uncertainty grows non-linearly with production, leading to increasing financial risk. At low pumping rates, cost predictions are relatively constrained. However, as production increases, both expected costs and uncertainty grow rapidly. This behaviour reflects the combined influence of aquifer response, well losses, and system hydraulics. The volume-price risk analysis provides a practical and quick tool for comparing cost to operational demand by linking production targets directly to financial risk. Rather than selecting a single "optimal" pumping rate, operators can evaluate trade-offs between production and the probability of exceeding budget constraints.

The current analysis assumes sufficient hydraulic capacity to meet target pumping rates. In practice, constraints such as pump performance, well interference, and aquifer limits would need to be included for full system evaluation. Additionally, the results are sensitive to the choice of prior distributions: poorly specified priors can bias posterior estimates, particularly when data are limited or noisy. The cost calculation depends on the assumed pump efficiency curve, which introduces an additional source of uncertainty not propagated through the current analysis. It should be noted that the posterior parameter distributions used in the synthetic example are derived from a single field well (CR-226) and are therefore site-specific; the absolute cost values presented are intended to be illustrative of the methodology rather than representative of a particular dewatering scenario.

Recommended Additional Analysis and Next Steps

Building on the methodology in Benson *et al.* (2026) and the synthetic evaluation presented here, several extensions are recommended to further improve applicability and robustness:

- Extend the framework to multi-well systems to capture interference effects and system-scale behaviour.
- Apply the method to field data and compare results with conventional deterministic approaches.
- Compare MCMC with alternative Bayesian methods (e.g., iterative ensemble smoothing) to evaluate trade-offs between computation cost and accuracy.
- Incorporate operational constraints such as pump capacity and aquifer limits.
- Integrate capital costs and energy price variability for more comprehensive economic evaluation.

Conclusions

Bayesian MCMC methods provide a robust framework for estimating aquifer and well parameters while accounting for uncertainty and parameter correlation. Parameter estimates derived from pumping tests are often highly correlated and non-unique, leading to non-Gaussian posterior distributions that cannot be adequately represented by single deterministic values. Application to a synthetic mine dewatering scenario shows that uncertainty in hydraulic parameters translates directly into uncertainty in energy consumption and operating costs. Both expected costs and their variability increase non-linearly with production. The volume-price risk analysis provides a practical means of linking pumping decisions to financial risk. The method is straightforward to implement, computationally efficient, and based on free, open-source Python tools. As such, it offers a practical and accessible approach for initial risk-informed dewatering planning. The ability to quantify the probability of meeting cost targets at different pumping rates, derived directly from pumping test data and requiring no commercial software, makes this framework well suited to early-stage mine planning where budget uncertainty has the greatest consequence.

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