

Modelling a Moving Disaster: A Novel Approach to Modelling Non-Newtonian Flow Post Tailings Dam Breach

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Abstract

Tailings dam failures cause catastrophic loss of life, environmental degradation, and economic damage. Contemporary industry-standard models typically rely on simplified solvers that fail to capture the non-Newtonian behaviour of tailings under rapid dynamic conditions, limiting hazard assessment accuracy and weakening emergency preparedness. This paper presents a finite-volume shock-capturing solver incorporating a quadratic viscoplastic rheology, an Osher-type approximate Riemann solver, and a hydrodynamic reconstruction technique preserving stationary and moving equilibria. Benchmarks against the idealised viscoplastic dam-break of Hungr and the 2019 Brumadinho disaster, show good agreement with published modelling results and observed data.

Keywords: Tailings flow, non-Newtonian, Riemann solver, Brumadinho failure

Introduction

When a tailings dam breaches, the released slurry behaves as a non-Newtonian fluid with a finite yield stress and shear-dependent resistance. This behaviour governs the sharp deceleration, progressive deposition, and localised arrest that define real events. Existing models struggle to represent this behaviour with sufficient fidelity for credible consequence analysis. Current approaches to tailings breach-runout modelling address only part of this challenge. Empirical volume-area relationships (Ghahramani *et al.*, 2020; Rico *et al.*, 2008) provide rapid screening but can be highly uncertain. Landslide models such as DAN3D and RAMMS use robust shock-capturing numerics yet represent tailings as purely frictional materials, rather than viscoplastic slurries (Sreekumar *et al.*, 2024). Shallow-water models, such as FLO-2D (FLO-2D Software Inc), MadFlow (Chen & Lee, 2003), and HEC-RAS (Brunner, 2016) implement adequate rheological formulations, however their diffuse numerical schemes tend to struggle to resolve sharp flow fronts, wet-dry transitions, and the steep depth gradients

(Kheirkhah Gildeh *et al.*, 2021). In fact, many implementations rely on non-conservative formulations that do not rigorously satisfy the conservation laws and lead to convergence towards incorrect solutions, as noted by Hou & Le Floch (1994). The practical consequence is that models may reproduce the gross inundation footprint whilst misrepresenting the arrival time and proximal tailings wave height. This is crucial for emergency planning and determination of contamination zones and downstream water quality impacts. This paper presents a finite-volume shock-capturing solver that combines a quadratic viscoplastic rheology with an Osher-type approximate Riemann solver and a well-balanced hydrodynamic reconstruction. The solver is validated against the idealised viscoplastic dam-break of Bryant *et al.* (1983) and benchmarked against Iber-NNF (Sanz-Ramos *et al.*, 2024) and DAN (Hungr, 1995). The Brumadinho 2019 tailings dam failure is then used as a case study to demonstrate the model's applicability to a real catastrophic event, with predictions compared against published modelling results.



Numerical model

To preserve the physical integrity of this moving disaster, the numerical scheme must resolve sharp fronts and rapid flow transitions without spurious oscillations. The mudflow is therefore modelled using the 2D depth-averaged incompressible flow equations in conservative form with a non-Newtonian source term:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S(U)$$

where $U = [h, hu, hv]^T$ are the conserved variables, $F = [hu, hu^2 + 0.5gh^2, huv]^T$ and $G = [hu, huv, hv^2 + 0.5gh^2]^T$ are the flux vectors, and $S(U)$ accounts for the source terms including bed-slope and non-Newtonian shear stress.

The rheological behaviour of the tailings is described by the O'Brien & Julien quadratic model (O'Brien *et al.*, 1985), in which the total shear stress is:

$$\tau = \tau_y + \eta \frac{du}{dy} + C \left(\frac{du}{dy} \right)^2$$

The three terms represent distinct physical contributions. The yield stress τ_y , is the finite stress required for flow to initiate. The viscous stress is characterized by the dynamic viscosity η . The turbulent-dispersive stress $C (du/dy)^2$ accounts for inertial and grain-collision forces, with coefficient $C = \rho_m l^2 + f(\rho_m, C_v) d_s^2$, where d_s is the representative particle diameter and ρ_m is the mixture density and C_v the concentration by volume. Collectively, the three stresses contribute to describe the full rheology of tailings behaviour from fluid-like behaviour at high shear rates near the breach, through viscous deceleration in the channel, to plastic arrest at the deposit boundary where the local stress falls below τ_y . The computational domain is discretized into a uniform Cartesian grid of rectangular finite-volume cells, over which the conserved variables are assumed piecewise constant within each cell. Intercell fluxes are computed using a generalised Osher-Solomon approximate Riemann solver (Dumbser *et al.*, 2010; Osher & Solomon, 1982). The solver evaluates the numerical flux by integrating the flux Jacobian along a straight-line path in phase space via three-point Gauss-Legendre

quadrature. This approach is entropy-satisfying and conservative, resolves sharp flow discontinuities and accurately captures the material interfaces and moving wet-dry fronts. The path-integral formulation avoids the need for explicit wave-speed estimates or flux limiters and remains well-defined across the full range of flow regimes encountered in a dam-break event. In addition, well-balanced treatment of the bed-slope source term is achieved through a hydrodynamic reconstruction (Berthon & Michel-Dansac, 2024) at cell interfaces, which ensures non-negativity of the mixture depth over irregular terrain and preserves dynamic and stationary equilibria. This property is essential for simulations on real topography, where bathymetric irregularities would otherwise generate spurious flows in dry or stationary regions. The full framework of the model is described in (Skifa *et al.*, 2026).

Test Cases

Idealized dam-break for viscous-plastic fluid

The numerical scheme is benchmarked against the semi-analytical one-dimensional viscoplastic dam-break problem on a horizontal plane of Bryant *et al.* (1983). Similar benchmarking exercises have been performed by many studies such as Hungr (1995) and Sanz-Ramos *et al.* (2024), which provides a semi-analytical reference solution for the runout of a finite volume of viscoplastic material released on a horizontal plane. This test case is widely used to assess the ability of depth-averaged solvers to reproduce the time evolution of flow depth and front position under purely viscoplastic resistance, without the complicating influence of topographic irregularities.

Tailings dam failure of Brumadinho

On January 25th, 2019, the B1 dam at the Córrego do Feijão iron ore mine near Brumadinho, Minas Gerais, Brazil (Figure 1), underwent a sudden and complete failure triggered by static liquefaction of the tailings material (Robertson *et al.*, 2019). The entire structure collapsed, releasing approximately 9.7 Mm³ of material, roughly 75% of the stored tailings volume, within five minutes (Robertson *et al.*, 2019). The resulting flow



Table 1 Key Parameters and values used in the model for Brumadinho mine tailings flow.

Parameter	Value
Mixture specific density ρ_m	26 kN/m ³
Yield stress τ_y	800 Pa
Dynamic viscosity η	100 Pa·s
Manning roughness n	0.167
Volumetric concentration C_v	55%
Roughness flow parameter K	2,285

travelled approximately 10 km downstream, inundating the mine’s canteen, swiping the railway bridge and reaching the Paraopeba river, ultimately claiming 270 lives. The outflow from the failed dam was a mixture of the embankment itself, stored tailings, and water, producing a hyperconcentrated non-Newtonian mudflow (Robertson *et al.*, 2019; Lumbroso *et al.*, 2021). Since no direct field or laboratory measurements of the Brumadinho tailings rheology are publicly available, the parameter values listed in Table 1 were derived from three complementary sources: (i) the Robertson *et al.* (2019) expert panel report, which provides estimates of in-situ volumetric concentration and material density; (ii) satellite imagery processed to delineate the final inundation extent; and (iii) parameter ranges reported in comparable numerical studies using MIKE21 (Lumbroso *et al.*, 2021), RiverFlow2D (Palu *et al.*, 2019) and TUFLOW HPC (TUTFLOW modelling team, 2021).

Unlike Lumbroso *et al.* (2021) and (Gibson *et al.*, 2022), who used a time-varying breach hydrograph as inflow, the present model adopted a dam-break initial condition, releasing the full tailings volume instantaneously at $t = 0$, under the assumption of an initial constant mixture free surface elevation of 960m with depths between 0 and 95m in the reservoir area. This is a more conservative assumption avoiding the additional uncertainty associated with breach parameter estimation. The computational domain was discretized into a regular grid based on a 12.5 m resolution ALOS PALSAR RTC DEM. The computational domain extent was obtained by adding a buffer of 200m of the observed inundation extent, which captures the flow from the dam toe to

the Paraopeba River including the Dam VI near the TSF dam.

Results and Discussion

Idealized dam-break for viscous-plastic fluid

Figure 2 presents the simulated flow depth profiles at $t = 0, 30, 60, 90,$ and 130 s alongside the Hungr (1995) analytical solution at arrest. The predicted final deposit extent, in Table 2, of 1883 m agrees with the Hungr (1995) analytical solution of 1892.8 m to within 0.5%, comparing favourably against DAN (1875 m) and Iber-NNF (1850 m).

The overprediction of proximal deposit depth relative to the analytical value of 7.36 m is attributed to the breakdown of the shallow water long-wave assumption in the near-dam region, where vertical accelerations are non-negligible, a known limitation shared by all depth-averaged viscoplastic solvers.

Tailings dam failure of Brumadinho

The results of the Brumadinho simulation are reported in Figure 3, showing the mudflow depth at 0 s, 90 s, 300 s, 600 s, 3600 s and 6600 s following the instantaneous dam-break. The results demonstrate that the solver captures the wave spread of the Brumadinho failure, though with limitations that are discussed below. The simulation reproduced the sequential inundation of three reference locations documented in post-event investigations. Predicted arrival times are compared against estimated arrival times from Lumbroso *et al.* (2021) in Table 3.

The flow reaches the canteen (black marker) by $t = 120$ s, consistent with video evidence and with Lumbroso *et al.* (2021), whose MIKE 21 Bingham simulation places canteen inundation within the first two

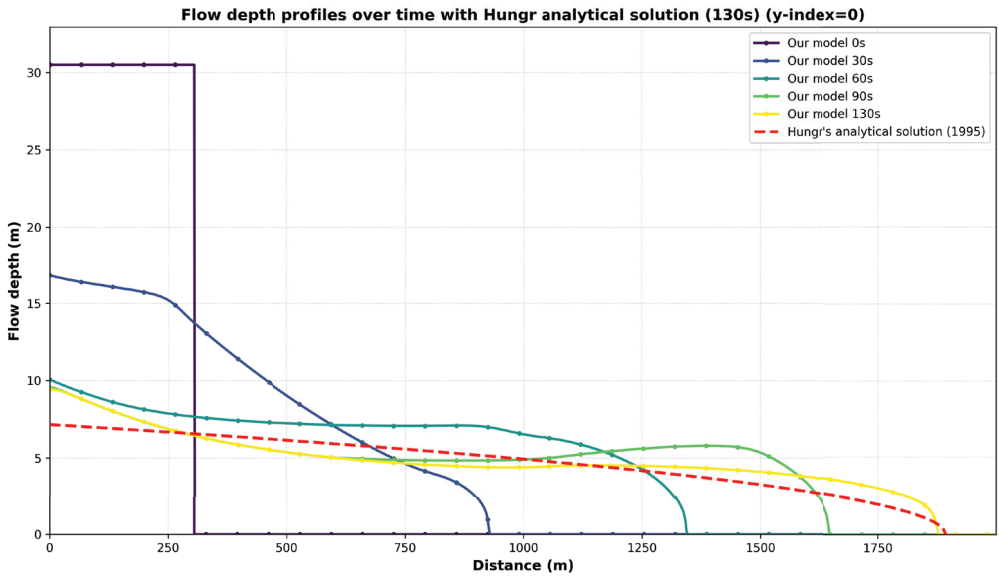


Figure 1 Comparison of the modelled mudflow depth and the observed mud extent. The back marker is the Canteen, and the red cross is the Railway bridge.

Table 2 Comparisons of final deposit extent between models.

Model	Final wave deposit extent (m)
Analytical solution	1892.80
Our model	1883.00
DAN	1875.00
Iber-NNF	1850.00

Table 3 Arrival times of observed and simulated flow at key locations.

Location	Estimated arrival time	Modelled arrival time
Mine canteen	≈ 2 min (observed)	2 min
Railway bridge	9 min 10 s	16 min 10 s
Paraopeba River	> 1.5 h	1 h 49 min 40 s

minutes of failure. The flow reaches the railway bridge (red marker) at $t = 970$ s (16 min 10 s). The estimated observed arrival time at the bridge, from video analysis by Lumbroso *et al.* (2021), is 9 min 10 s. The later predicted arrival in the present simulation, despite the dam-break initial condition producing a faster initial wave celerity than the time-varying hydrograph used by Lumbroso *et al.* (2021), could be attributed

to the additional resistance introduced by the turbulent-dispersive stress term $C(du/dy)^2$ of the quadratic model, which is absent from the Bingham formulation used in Lumbroso *et al.* (2021) and becomes significant at the high shear rates in the confined valley reach between the canteen and the bridge. At $t = 6580$ s the modelled flow has reached the Paraopeba River (yellow marker), approximately 10 km downstream. Lumbroso *et al.* (2021) report

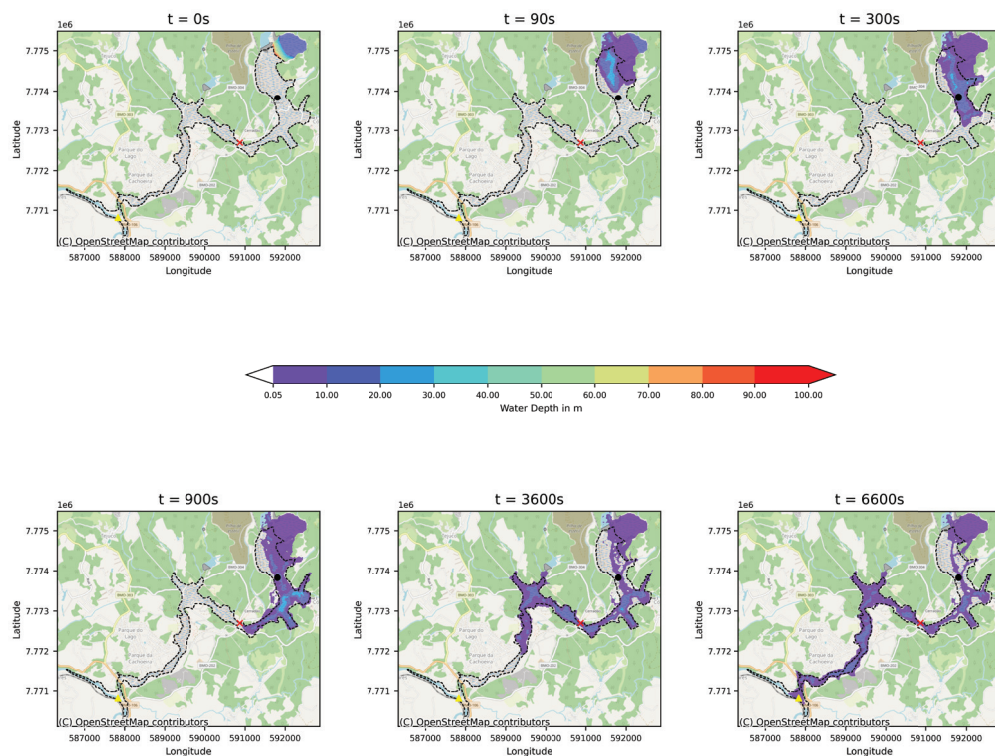


Figure 2 Comparison of the modelled mudflow depth and the observed mud extent. The back marker is the Canteen, the red cross is the Railway bridge, and the yellow triangle is the river.

a modelled river arrival of 1 h 26 min 05 s against an estimated observed arrival of approximately 1 hour 30 min to 2 hours 30 min (TUTFLOW modelling team, 2021). The small overprediction of runout arrival time in the present simulation is compounded by two factors not reported in other studies: a portion of the simulated volume was diverted into a neighbouring reservoir, reducing the volume propagating toward the river, a behaviour consistent with satellite imagery showing tailings reaching the adjacent reservoir, and the use of Bingham-calibrated parameters in a quadratic formulation introduces structural parameter mismatch. The dam-break initial condition was adopted on physical grounds, as documented in video evidence and confirmed by Robertson *et al.* (2019) of the of the Feijão dam that failed by near-instantaneous liquefaction across the full reservoir, making this assumption a physically appropriate representation of the mobilisation mechanism rather than a

simplifying convenience. Whilst Lumbroso *et al.* (2021), Gibson *et al.* (2022), Ligier (2020), and Raman & Liu (2019) adopted time-varying hydrographs, they were not directly measured but derived from geotechnical breach models requiring several input parameters, dam geometry, erodibility, and pore pressure distribution, that are not publicly available for Brumadinho failure, introducing a new source of uncertainty. The dam-break assumption avoids this uncertainty whilst remaining consistent with the observed failure mode. A systematic comparison between both approaches would help isolate any sensitivity of predicted arrival times and runout extent to the initial conditions.

Conclusion

The results presented in this paper demonstrate that the proposed finite-volume solver, reproduces the essential physical behaviour of a large-scale tailings dam failure.



The solver is numerically stable, conservative, and correctly enforces the yield criterion throughout the domain, producing the sharp flow fronts and abrupt flow arrest that are characteristic of tailings materials. The Brumadinho case study demonstrates that a physically consistent numerical model is a necessary condition for reliable tailings flow predictions. Without correct enforcement of the shock-capturing numerics and well-balanced treatment of topography, the rheological parameters cannot be correctly translated into physically consistent flow behaviour. Moreover, rheological parameters are a dominant source of uncertainty in tailings breach modelling, with runout distance, arrival time, and inundation extent all shown to be strongly sensitive to yield stress and viscosity (Lumbroso *et al.*, 2021; Palú and Julien, 2019). Several limitations and directions for future work are identified. The present model does not account for breach initiation or erosion-progressive failure, and the 2D depth-averaged formulation neglects the vertical flow component; extension to a multilayer framework is therefore identified as a future priority. In addition, the sensitivity of the model to rheological parameters needs to be quantified, future work will conduct a formal sensitivity and uncertainty analyses to produce probabilistic intervals on predicted inundation extents. Addressing these limitations will enable better determination of inundation and contamination zones and downstream water quality impacts.

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