# NEW CONCEPT TO DESCRIBE FLOW THROUGH POROUS MEDIA

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ABSTRACT : The present paper introduces a new treatment of the ground water flow. The velocity distribution in the pores is always changing stochastically, but the mean values show regularity. The velocity field of the flow through porous media is taken into consideration as a mean velocity field, ant the fluctuation about the mean, superimposed to it. In this base the equation of motion can be derived.

RESUME : La présente conférence introduit un nouveau traitement du débit de l'eau souterraine. La distribution de la vitesse dans les pores est toujours changée stochastiquement, mais les valeurs principales montrent la régularité. Le champ de vitesse du courant à travers le milieu poreux est pris en considération comme une moyenne de champs de vitesse, et la fluctuation autour de cette valeur qui lui est superimposée. A partir de cette base, l'équation du mouvement sera démontrée.

RESUMEN : Esta comunicación presenta un nuevo tratamiento del flujo del agua subterránea. La distribución de velocidad en los poros cambia siempre estocásticamente, pero los valores medios muestran regularidad. El campo de velocidades del flujo, a través de medios porosos, se considera como una media de los campos de velocidad, a la que se superponen las fluctuaciones sobre dicha media. Con estas premisas se deduce la ecuación del movimiento.

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To describe the movement of ground water a generally accepted method is known. Assuming laminar flow of fluid through porous media, general flow equations of ground water can be derived from Darcy's law and the equation of continuity. In some cases the equations for steady flow of a single fluid can be simplified into the Laplace equation, for which we can obtain solution by potential-theory methods. This is the way which is mostly followed.

This study has been motivated by three factors. First from a practical point of view, the new mine-opening works in Hungary are done in water dangerous areas. The classical preventive methods seem here insufficient. The new so-called instantaneous prevention technique against water intrusion in mines requires a better knowledge of the nature of ground water movement.

Secondly, during laboratory experiments the relationship between the flow rate and head loss was obtained non-linear, though the motion had been unquestionably laminar. Thus a non-linear resistance law occurs as soon as the inertia terms are taken into account, without the fluid being in true turbulent motion. Non-linearity can be found at very low Reynolds number /such $\approx$ 0,1/.

Thirdly, to obtain a comprehensive view of flow through porous media, the balance equations of the transport theorem present themselves as the most advanced means. In spite of the partial success with the explanation of Darcy's law, we introduce a really dynamic description of seepage flow upon a general ground.

Our basic assumptions are the following. The detailed fine structure of the rock and the fluid in its pores is replaced by a continuous model of matter having appropriate continuum properties so defined as to ensure that as the macroscopic scale the behaviour of the model duplicates the behaviour of the real material. We assume that the pores would be ignored individually if the pore sizes are rather small compared with the dimensions of the problem. The two-component /rock and fluid/ system forms a complex continuum. The essential mathemathical simplification of the continuum model is that the average properties in the infinitesimal volume dV surrounding point P are assigned in the limit to the point P itself. Thus one arrives at an equivalent but fictious continuous material, locally homogeneous and unstructured in which the real material is represented by continuously distributed field functions of its phisical properties: the density, velocity, stress, etc.

As an illustration of the limiting process by which the local continuum properties are defined, imagine a small volume $\Delta V$ surrounding a point P, and let $\Delta V_p$ be the pore volume of material in $\Delta V$. Now the ratio $\Delta V_p / \Delta V$ will except as noted below, asymptote toward some limit as $\Delta V$ is made smaller. When $\Delta V$ reaches some value dV, which may be thought of as the infinitesimal volume of the complex continuum, significant random fluctuations begin to appear because the number of pores in $\Delta V$ is too small, and the notion of the ratio $\Delta V_p / \Delta V$ then loses practical utility. Thus the porosity at the point P is defined as

$$\Pi = \lim_{\Delta V \to dV} \frac{\Delta V_p}{\Delta V} = \frac{dV_p}{dV} \qquad /1/$$

in which the limiting process is carried only to dV with the understanding, however, that $\Pi$ is associated with the point P itself. Thus we introduced a continuous scalar field of porosity $\Pi/\vec{r},t/$.

Assuming that $\Pi/\vec{r},t/$ is a continuous field and the pores are filled perfectly with fluid, we get for mass of fluid per unit volume

$$\beta = \Pi\,\rho \quad , \qquad /2/$$

where $\rho$ is the density of the fluid.

Let the local velocity of the fluid be designated by the vector $\vec{v}$. This is the velocity with respect to fixed coordinates. It is not the velocity of the fluid particles in the pores, but rather the average velocity with which the particles of fluid in a small region are moving. If the infinitesimal volume dV is rather small compared with the investigated domain of the flow, $\vec{v}/\vec{r},t/$ can be considered as a continuous vector field.

The principle of conservation of mass will be expressed as

$$\frac{\partial}{\partial t}\,/\Pi\,\rho\,/ + \mathrm{div}\,/\Pi\,\rho\,\vec{v}/ = j \qquad /3/$$

for the unit volume of the complex continuum.

This statement means that the rate of increase of fluid mass within the unit volume of complex continuum, and the rate at which fluid mass crosses into it, over its surface, are equal with the rate at which fluid mass is produced /or disappeared/ within the unit volume by diffusion, adsorption or chemical reaction.

Without mass production the right side of the equation is vanished:

$$\frac{\partial}{\partial t}\,/\Pi\,\rho\,/ + \mathrm{div}\,/\Pi\,\varrho\,\vec{v}/ = 0. \qquad /4/$$

For perfectly rigid rocks the porosity isn't depend on time, it varies only with position, thus

$$\Pi \frac{\partial \varrho}{\partial t} + \text{div} / \varrho \, \Pi \, \vec{v} / = 0. \qquad /5/$$

The product $\Pi \vec{v}$ is called as the local seepage velocity:

$$\vec{q} = \Pi \vec{v}. \qquad /6/$$

In an incompressible flow the density $\varrho$ varies neither time, nor position, continuity equation becomes very simple form:

$$\text{div } \vec{q} = 0. \qquad /7/$$

The flow in each pore-channel can be described by the equation of continuity and by the Navier-Stokes equation. Nevertheless this is not an easy way to solve any problems. Geometry of pore-system is of a wide variety of structures. If it were possible to prescribe the boundary conditions for only one pore-channel, the solution would be difficult and the results would have no any prectical utility. The full description of the flow in the pores requires an enormous number of equations, and the solution of these equations depends strongly on minute details of the initial and boundary data, while these details are never available in practice.

If the real in situ motion in the pores is replaced by a space--averaged flow, we get suitable means to solve practical problems. For mathematical treatment it is convenient to split up the velocity of the pore flow $\vec{u}$ into a mean $\vec{q}$, and a fluctuation $\vec{w}$ superimposing on it.

$$\vec{u} = \vec{q} + \vec{w}, \qquad /8/$$

where q is the local seepage velocity. Let's assume that the fluid is incompressible, and the velocity field u/r,t/ satisfies the Navier Stokes equations, and the equation of continuity:

$$\frac{\partial \vec{u}}{\partial t} + /\vec{u} \bullet \vec{u}/\nabla = \vec{g} - \frac{1}{\varrho} \nabla p + \nu \nabla^2 \vec{u} \qquad /9/$$

$$\nabla \vec{u} = 0 \qquad /10/$$

Substituting Eq.8. into Eq.9. and following the usual rules for averaging we obtain

$$\frac{\partial \vec{q}}{\partial t} + /\vec{q} \bullet \vec{q}/\nabla + \overline{/\vec{w} \bullet \vec{w}/\nabla} = \vec{g} - \frac{1}{\varrho} \nabla p + \nu \nabla^2 \vec{q} \qquad /11/$$

From this equation the following important facts are obtained.

The rate of increase of momentum is composed of three terms. The first term is the rate of increase of momentum within the unit volume, the second is the rate of momentum crosses into it over its surface with velocity $\vec{q}$, the third is the rate of momentum crosses over the surface of the unit volume by the fluctuations $\vec{w}$. The latter term is an irreversible transport of momentum, which leads to an intensive energy dissipation.

The righ side of the equation consists of the sum of external forces acting on the fluid within the unit volume.

It is convenient to write the equation of motion in such a way that on the left side of it the rate of momentum will be stood, which refers to the seepage velocity field $\vec{q}$.

$$\frac{\partial \vec{q}}{\partial t} + /\vec{q} \bullet \vec{q}/\nabla = \vec{g} - \frac{1}{\varrho} \nabla p + \nu \nabla^2 \vec{q} - \overline{/w \bullet w/}\, \nabla \qquad /12/$$

In this aspect the latter term can be considered as an added external force, expressing the interaction between the porous medium and the fluid.

This force acts on the fluid against the flow, and it can be expressed by the divergence of a virtual stress tensor:

$$\overline{/-\vec{w} \bullet \vec{w}/\nabla} = \frac{1}{\varrho}\, \mathrm{Div}\, /-\varrho\, \overline{\vec{w} \bullet \vec{w}}/ = \frac{1}{\varrho}\, \mathrm{Div}\, \underline{\underline{S}} \qquad /13/$$

As it is shown, the virtual stress tensor is simmetric:

$$\underline{\underline{S}} = -\varrho \begin{bmatrix} \overline{w_x w_x} & \overline{w_x w_y} & \overline{w_x w_z} \\ \overline{w_y w_x} & \overline{w_y w_y} & \overline{w_y w_z} \\ \overline{w_z w_x} & \overline{w_z w_y} & \overline{w_z w_z} \end{bmatrix} \qquad /14/$$

therefore six unknown scalar function occur in the equation of motion.

In order to be able to make calculations of seepage flow it is necessary to find any expression for the additional virtual stresses, which relates them to the averaged quantities. Firstly we should assume the existence such a relationship, secondly we must look for this relationship in any reasonable form like the constitutive equation, moreover it should be in accordance with the Onsager's law. If this assumed expression substituting to the equation of motion leads to results corresponding with experimental data, it can be accepted as a confirmed relation.

As an illustration of the foregoing, an experimentally proved relationship, the Darcy's law will be derived.

As it is known the flow rate through porous media is proportional to the head loss and inversely proportional to the length of the flow path. The experimental verification of Darcy's law can be performed with water flowing at a rate Q through a cylindrical tank of radius R, packed with sand and having pierometer taps a distance L apart, as shown in Fig. 1.

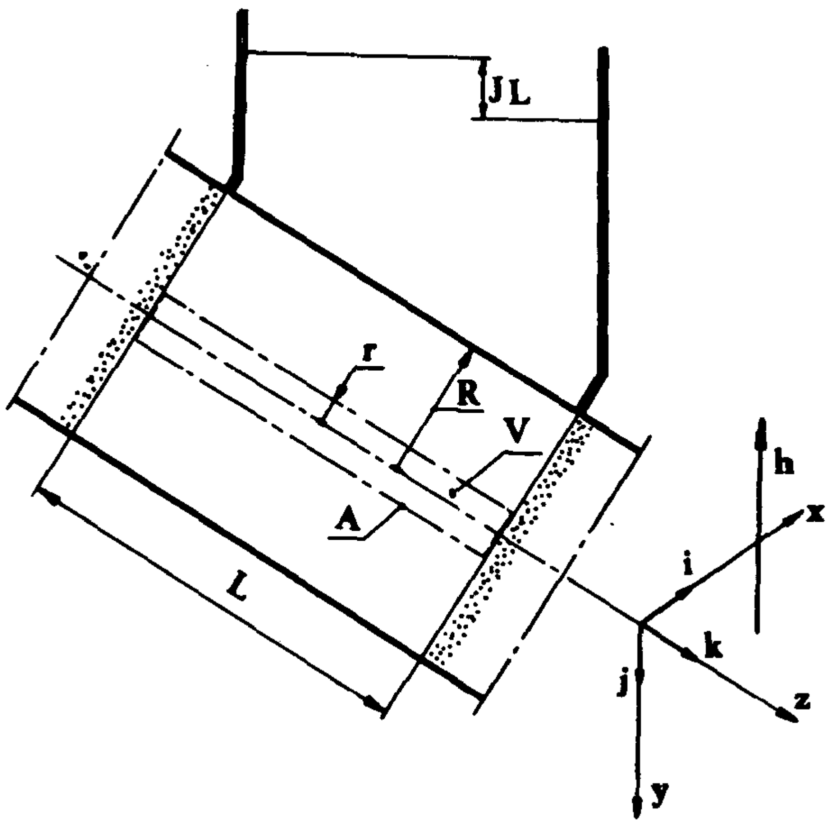


Figure 1.

Let's consider a coaxial cylindrical control volume of radius r, to which we write the equation of motion in the integral form:

$$\int_V \frac{\partial /\varrho \vec{q}/}{\partial t} dV + \int_{/A/} \varrho \vec{q} \,/\vec{q} d\vec{A}/ = \int_V \varrho \vec{g} dV - \int_{/A/} p d\vec{A} +$$

$$+ \int_{/A/} 2 \mu \,/\vec{q} \bullet \nabla + \nabla \bullet \vec{q}/ \, d\vec{A} - \int_{/A/} \varrho \,\overline{/\vec{w} \bullet \vec{w}/} \, d\vec{A} \qquad /15/$$

This equation can be simplified, if the following assumptions are taken to consideration:

/i/ The velocity field has only axial component

$$q = q_z; \qquad q_x = q_y = 0$$

/ii/ The equation div $\vec{q} = 0$, will be

$$\frac{\partial q_z}{\partial z} = 0$$

/iii/ The flow is steady:

$$\frac{d\vec{q}}{dt} = 0$$

Thus the left side of the equation of motion will be vanished. Assuming the existence of a force potential, we can write:

$$0 = -\int_V \varrho \nabla \left(U + \frac{p}{\varrho}\right) dV + \int_{/A/} 2\,\mu\,/\vec{q}\bullet\nabla + \nabla\bullet\vec{q}/\; d\vec{A} -$$

$$- \int_{/A/} \varrho\, \overline{/\vec{w}\bullet\vec{w}/}\; d\vec{A} \qquad /16/$$

In gravitational field

$$U = gh,$$

thus we get:

$$-\vec{k}\int_V \varrho\, g \frac{\partial}{\partial z} /h + \frac{p}{\gamma}/\; dV = \varrho\; g\; J\; r^2 \pi\; L\, \vec{k}\,,$$

in which J the hydraulic gradient:

$$J = -\frac{\partial}{\partial z} /h + \frac{p}{\gamma}/ = \frac{h_1 - h_2}{L} + \frac{p_1 - p_2}{\gamma L} \qquad /17/$$

We assume, that Stokes's principles can be generalized to the seepage velocity field. The linear nature of Darcy's law affirms this.

At least we assume that the tensor field $\overline{\vec{w}\bullet\vec{w}}$ is axially symmetric, and don't vary in axial direction.

Thus the integrands are constant on the surface of the control volume and we get the following expression

$$g \, J \, r^2 \pi \; L \vec{k} + \mu \frac{dq}{dr} 2r \, \pi \; L \vec{k} - \varrho \, \overline{w_r \, w_z} \, 2 \, r \, \pi \; L \vec{k} = 0 \quad /18/$$

For the unit volume it is obtained

$$\frac{g \, J \, r}{2} + \pi \frac{dq}{dr} - \varrho \, \overline{w_r \, w_z} = 0 \quad /19/$$

Now we must look for some reasonable assumption in accordance with Onsager's law for the only element of the virtual stress tensor:

$$- \varrho \, \overline{w_r \, w_z} = \Lambda \frac{dq}{dr} , \quad /20/$$

where $\Lambda$ is the conduction coefficient of the momentum, which characterizes the interaction between the rock and fluid. Generally it is not a constant quantity, but we feel strong temptation to consider $\Lambda$ to an additional viscosity. This simplification is suitable to derive Darcy's law, but the measurements of the local seepage velocity lead to that result that $\Lambda$ is not constant along the radius, it depends on the radius, and the inhomogenity of the velocity field:

$$\Lambda = \Lambda \, /r \, \frac{dq}{dr}/.$$

The simplest possibility is $\Lambda =$ const, for which we must solve the following differential equation:

$$\frac{g \, J \, r}{2} + / \, \mu + \Lambda \, / \frac{dq}{dr} = 0 \quad /21/$$

After integration we get:

$$q = - \frac{g\ J}{4\ /\mu + \Lambda\ /}\ r^2 + C \qquad /22/$$

The determination of the constant is not too easy, since the seepage velocity is not a real, but a virtual velocity. Thus we can't prescribe, that the tangential component at the wall will be vanished. If we choose to boundary condition

at $r = R$, $q = 0$, the flow rate is smaller than the measured value. This shows that the local seepage velocity at the $r = R$ will not be zero, the seeping fluid "slips" on the wall. Thus the constant must be determined from the flow rate:

$$Q = 2\pi \int_0^R q\ r\ d\ r \qquad /23/$$

After substitution we obtain

$$Q = - \frac{\pi\ g\ J\ R^4}{8\ /\mu + \Lambda\ /} + C\ R^2 \pi \qquad /24/$$

Measuring the flow rates and the corresponding hydraulic gradient values, the $\Lambda$ coefficient and the constant C /the local seepage velocity on the wall/ can be determined.

Since for a given experimental apparatus only Q and J varies, the relationship between the flow rate and the hydraulic gradient is

$$Q = K\ /J - J_o/ \qquad /25/$$

where $J_o$ is an additional constant, which is a threshold hydraulic gradient at which the seepage motion will be started.

Thus we derived the Darcy's law on the base of continuum mechanics. This can be a possible proof of the introduced description of the seepage flow. The basic equations derived above will be applied subsequently to obtain analytic or numerical solutions to particular seepage flow problems.

REFERENCES

[1] ASSZONYI, CS. and RICHTER, R.: The Rheological Theory of Rock Mechanics./in Hungarian/
NIMTK, Budapest /1975/.

[2] ERINGEN, C.A.: Nonlinear Theory of Continuous Media. Mc Graw Hill, New York /1962/.

[3] GOLDSTEIN, S.: Lectures on Fluid Mechanics Interscience Publishers. New York /1960/.

[4] LAMB, H.: Hydrodynamics.
Dover Publications, New York /1945/.

[5] ONSAGER, L.: Phys. Rev. 37, 405 /1931/.

[6] PRIGOGINE, I.: Étude thermodynamique des phénomenes irrevérsibles.
Dunod-Dezoer, Paris /1947/.

[7] TRUESDELL, C. and TOUPIN, R.A.: The Classical Field Theories, Handbook of Physics, Vol III./1. Springer, Berlin /1960/.