RELIABILITY AND THE FACTOR OF SAFETY DUE TO PIPING

Harr, M. E. and Sipher, D. J.

ABSTRACT : A hybrid finite element program was used to evaluate the effect of the randomness and the uncertainty of the coefficient of permeability on the exit gradient. A log normal distribution was used to assign permeabilities to the elements of the finite element grid. It was found that the distribution of exit gradients could be approximated by a normal distribution having a mean value approaching that of a perfectly homogeneous soil. An upper bound of the coefficient of variation of the true exit gradient was obtained from a 99 % confidence interval of the chi-square distribution. This coefficient of variation was used with the theoretical solution of confined sheet pile flow to determine the probability of piping (i. e., F. S.) piping < 1.0), which corresponds to an exit gradient greater than 1.0.

RESUME : Une étude, combinant les techniques d'éléments fins et de probabilité, a été effectuée pour évaluer l'effet de l'incertitude et de la variabilité associé au coefficient de perméabilité, sur le gradient hydraulique de sortie. Une distribution logarithmique normale a été utilisée pour assigner des perméabilités aux éléments du modèle. On a trouvé que la distribution des gradients de sortie peut se rapprocher d'une distribution normale, avec une moyenne similaire à celle d'un sol parfaitement homogène. Une limite supérieure du coefficient de variation du gradient de sortie a été obtenue pour une limite de confiance de 99 %, d'une distribution chi-carrée. Ce coefficient de variation a été utilisé ainsi que la solution théorique au problème de l'écoulement confiné autour du pilotage pour déterminer la probabilité que le phénomène de siphonnement se produise.

RESUMEN : Combinando las técnicas de elementos finos y de probabilidad, se ha efectuado un estudio para evaluar el efecto de la incertidumbre y variabilidad asociada con el coeficiente de permeabilidad, sobre el gradiente hidráulico de salida. Se ha utilizado una distribución logarítmica normal, para asignar permeabilidades a los elementos de la malla del modelo. Se encontró que la distribución de gradientes de salida, se puede aproximar con una distribución normal, con una media similar a la de un suelo perfectamente homogeneo. Se obtuvo un límite superior del coeficiente de variación del gradiente de salida, para un límite de confianza de 99 %, de una distribución chi-cuadrado. Este coeficiente de variación fue utilizado junto con la solución teórica al problema de flujo confinado alrededor de un tablaestacado, para determinar la probabilidad de que el fenómeno de sifonamiento ocurra.

School of Civil Engineering, Purdue University
West Lafayette, Indiana 47097. U. S. A.

INTRODUCTION

The classical approach for the transmission of boundary energy through soil bodies has been to consider soil as a homogeneous, continuous medium. Such assumptions have permitted the discovery of deterministic solutions to many soil mechanics problems. When dealing with seepage problems the soil body was considered to be a continuous medium of many interconnected openings which serve as the fluid carrier. It has long been recognized that even for uniform spheres and ideal packings these interconnected openings are not regular but consist of cavernous cells interconnected by narrower channels. As natural soils contain particles which deviate considerably from a spherical shape and from uniform size, natural pore channels defy rational description.

In spite of the obvious uncertainties involved, important classes of seepage problems have been solved using a macroscopic deterministic approach. Fundamental to this approach is the assumption that the soil is homogeneous and that it has the same permeability at each point. These assumptions lend themselves readily to theoretical solutions.

What if the permeability is not the same at each point? Are the theoretical solutions still valid? Are the curves that define flownets still smooth and regular? This paper will investigate such questions with respect to one important class of seepage problems, i.e. the exit gradient for a system of confined flow exhibiting a downstream cut off.

PIPING

Piping is a consequence of the internal erosion of a soil mass beneath a water retaining structure. Piping is initiated when, at some point along the downstream soil surface, the weight of a submerged soil particle is acted upon by an unbalanced seepage force. In effect an energy transfer takes place. The total energy dissipated is that associated with the difference in water elevations from one side of a structure to the other. This energy transfer is called the head loss, and the head loss per unit length is the hydraulic gradient. The seepage force (Harr, 1962) can then be expressed as $i\gamma_w$ where i is the gradient. Equating this to the submerged unit weight of the soil results in a measure of the critical hydraulic gradient required for piping to begin. This gradient, i_{cr}, is approximately equal to unity. In general,

$$i_{cr} = (S_s-1)/(1+e) \tag{1}$$

The factor of safety with respect to piping is the ratio of the critical gradient to the actual gradient along the discharge surfaces.

THEORY

The theoretical solution for the exit gradient for a vertical cut off, such as a sheet pile (see Figure 1) is (Harr, 1962)

$$I_e = \frac{h\pi}{4KTm}(1-m^2t^2)^{\frac{1}{2}} /m't \quad (2)$$

where I_e = exit gradient
- T = thickness of permeable layer
- h = water head
- K = complete elliptic integral of the first kind of modulus, m
- m = $\sin\pi s/2T$

and m' = $\cos\pi s/2T$

where s = the depth of embedment of the sheetpile
- t = $(\cot\pi s/2T)(\tan^2\pi x/2T + \tanh^2\pi s/2T)^{\frac{1}{2}}$

where x = horizontal distance from the sheetpile

For the considered section the maximum gradient occurs next to the sheetpile ($x = 0$) and is given as

$$I_e max = h\pi/4KTm \quad (3)$$

It is standard procedure to employ (Khosla et al, 1954) a factor of safety ranging from 4 to 7. The lower value is for gravels and the upper value for fine sands.

COMPUTER MODELING

A finite element computer program was selected to model real soils, which can be highly variable. This analytical tool allows one to accomodate for variations in soil properties from point to point. The elements of a finite element grid can be thought of as representing small units of homogeneous soil that when grouped together represent a non-homogeneous mass.

As the resulting exit gradients were obtained for finite elements rather than at specific points along the discharge surface, a standard procedure for plotting the results had to be established. To do this, the vertical exit gradient for each element was considered to act along the vertical edge of the element (see Figure 2).

For s/T values in the range of interest, numerical values given by the finite element solution were found to be within approximately 10% of those given by the exact theoretical solution Equation (3) (see Figure 2), for homogeneous soils.

Initially a region of dimensions T by 3T was selected as shown in Figure 3a.

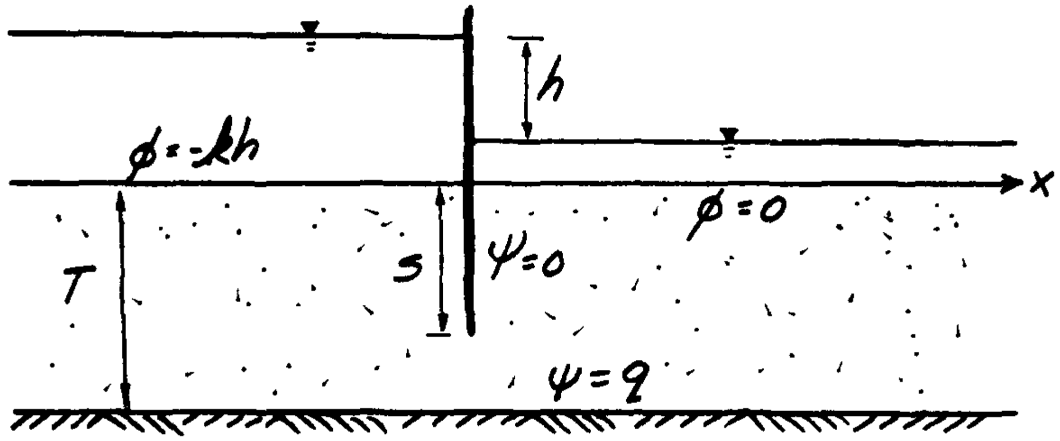
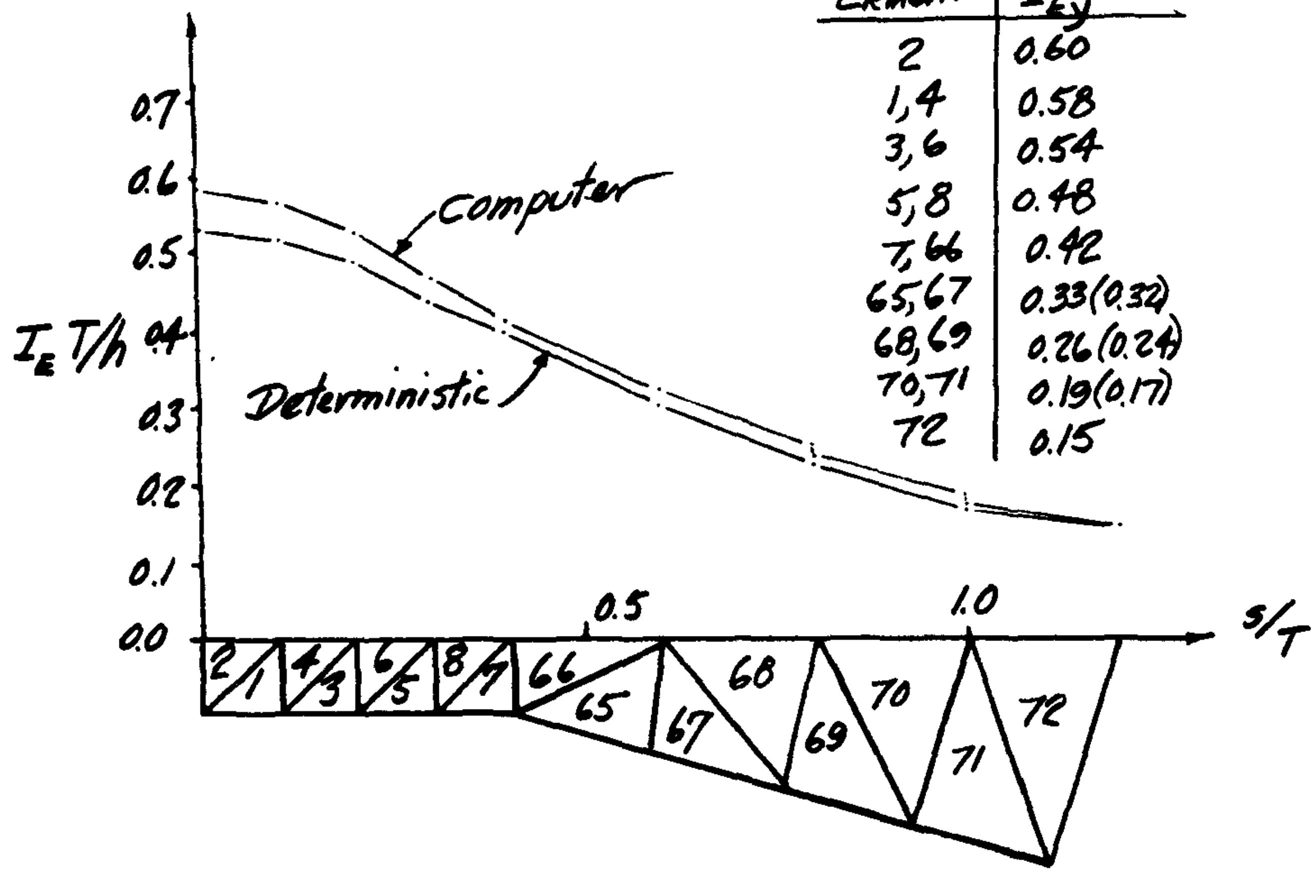


Figure 1: Geometric and Boundary Conditions

Figure 2: Computer vs. Deterministic Results



Element	I_{Ey}
2	0.60
1,4	0.58
3,6	0.54
5,8	0.48
7,66	0.42
65,67	0.33(0.32)
68,69	0.26(0.24)
70,71	0.19(0.17)
72	0.15

with the noted boundary conditions. This grid proved to be too large for efficient computer use and was reduced to the size shown in Figure 3b. The boundary conditions used along the right boundary of the new grid were taken to be the nodal values of that surface for the homogeneous conditions of the larger grid. A sensitivity analysis was performed which indicated that large changes in these boundary values had but very little influence on the exit gradients near the sheetpile.

To assign random permeabilities to the elements of the grid, a mean value of the coefficient of permeability was multiplied by a random number obtained from a lognormal distribution and assigned to an element. A lognormal distribution allows permeabilities to range from zero to infinity, thus representing soil units from a boulder to a complete void. The resulting element coefficients of permeability displayed three orders of magnitude.

RESULTS

The first computer run produced the results shown in Figure 4. This illustrates the degree of deviations that can be expected from exact theoretical solutions for even a slightly non-homogeneous soil mass.

100 runs were made for a ratio of $s/T = 0.55$. The coefficient of permeability was taken to be log-normal with $\ln k = N(0,1)$. The resulting exit gradients and their characteristics for each element on the discharge surface are summarized in Table 1. The element numbers are keyed to Figure 2. The mean values of the exit gradients for each element are also plotted with their corresponding deterministic values in Figure 5. The finite element solution is seen to conform quite well to the closed-form solution.

The statistical data in Table 1, the coefficients of skewness and kurtosis, β_1 and β_2, indicate that the distribution of exit gradients may be approximated by a normal distribution $\beta_1 = 0$, $\beta_2 = 3.0$ without undue error.

The coefficients of variation of the exit gradients are seen, Table 1, to vary from 34% to 44% for the elements sampled. A chi-squared analysis for a one-tailed 99% confidence limit gave an upper limit value of 45%.

PROBABILITY OF PIPING

With the distribution of the exit gradient taken to follow a normal distribution, the probability can easily be had of it exceeding its critical value; that is, the probability of piping. This is shown schematically on Figure 6 as the shaded area.

Figure 7 shows that the probability of piping decreases rapidly with increasing distance from the piling. This figure also indicates the contribution of the thickness of the layer to the security against piping. As noted, the region of insecurity is confined to a lateral distance from the piling of 0.2T, where T is the thickness of the permeable layer.

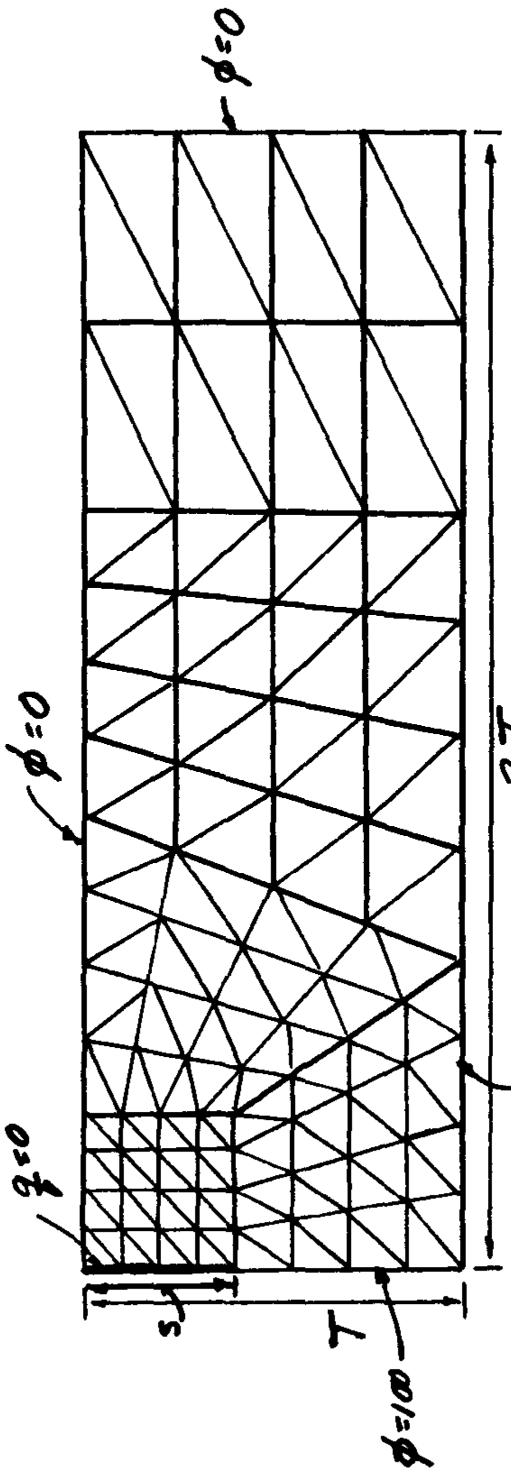
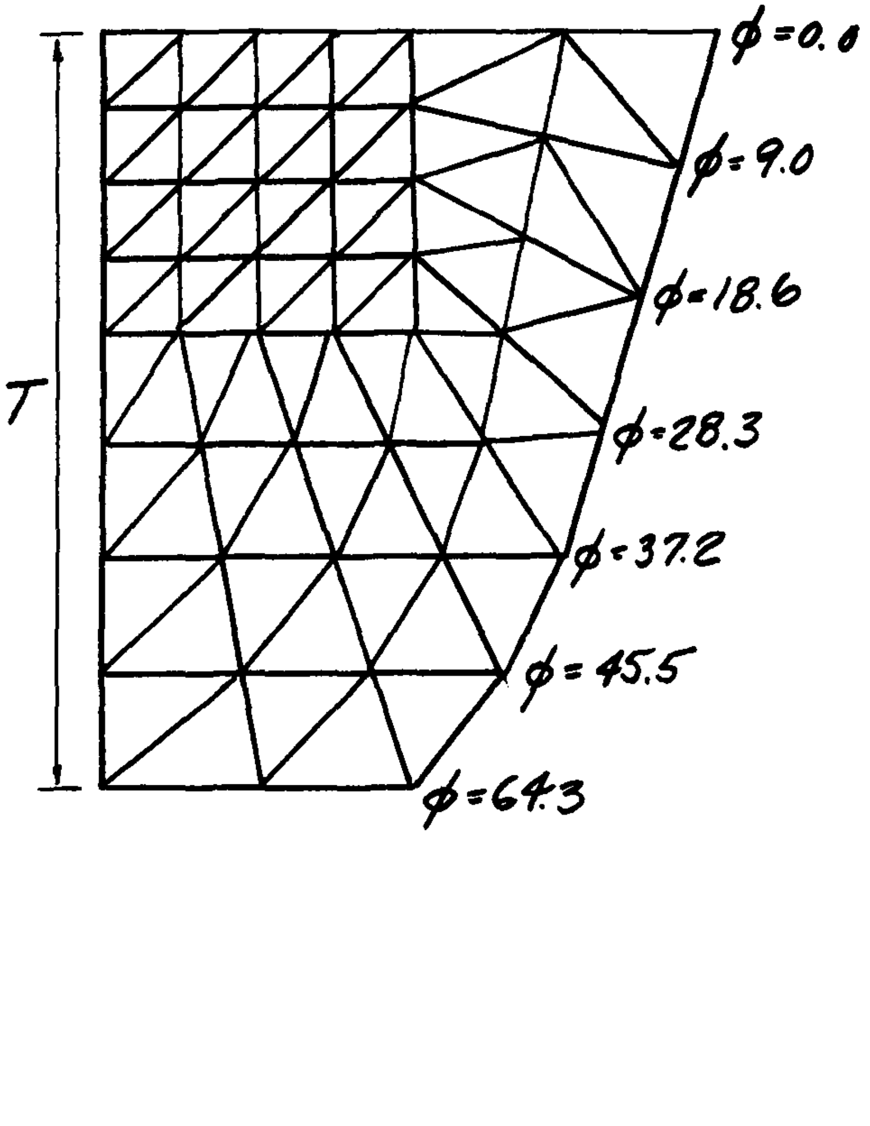


Figure 3a : Finite Element Grid

Figure 3b: Finite Element Grid



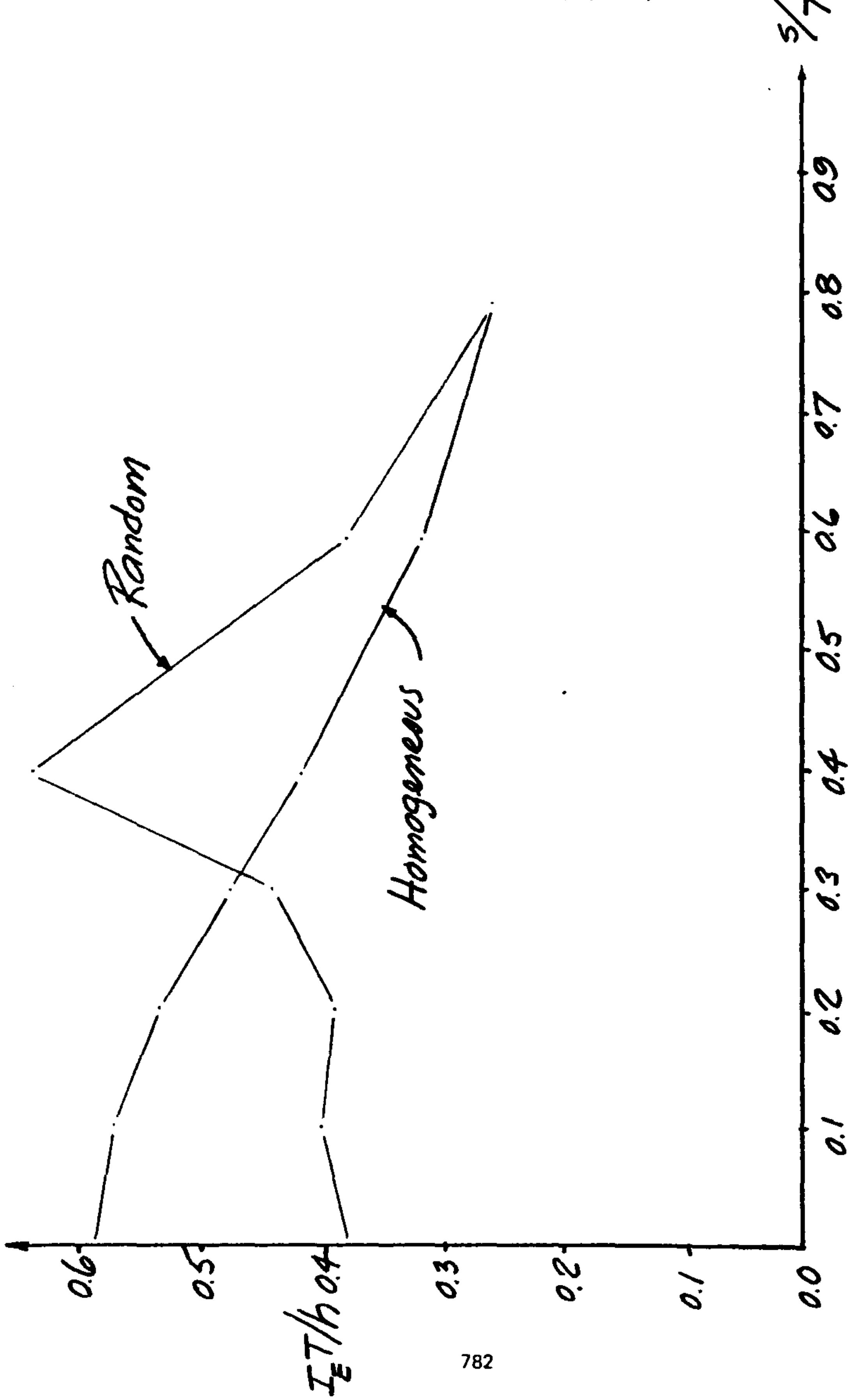


Figure 4: Single Random Run

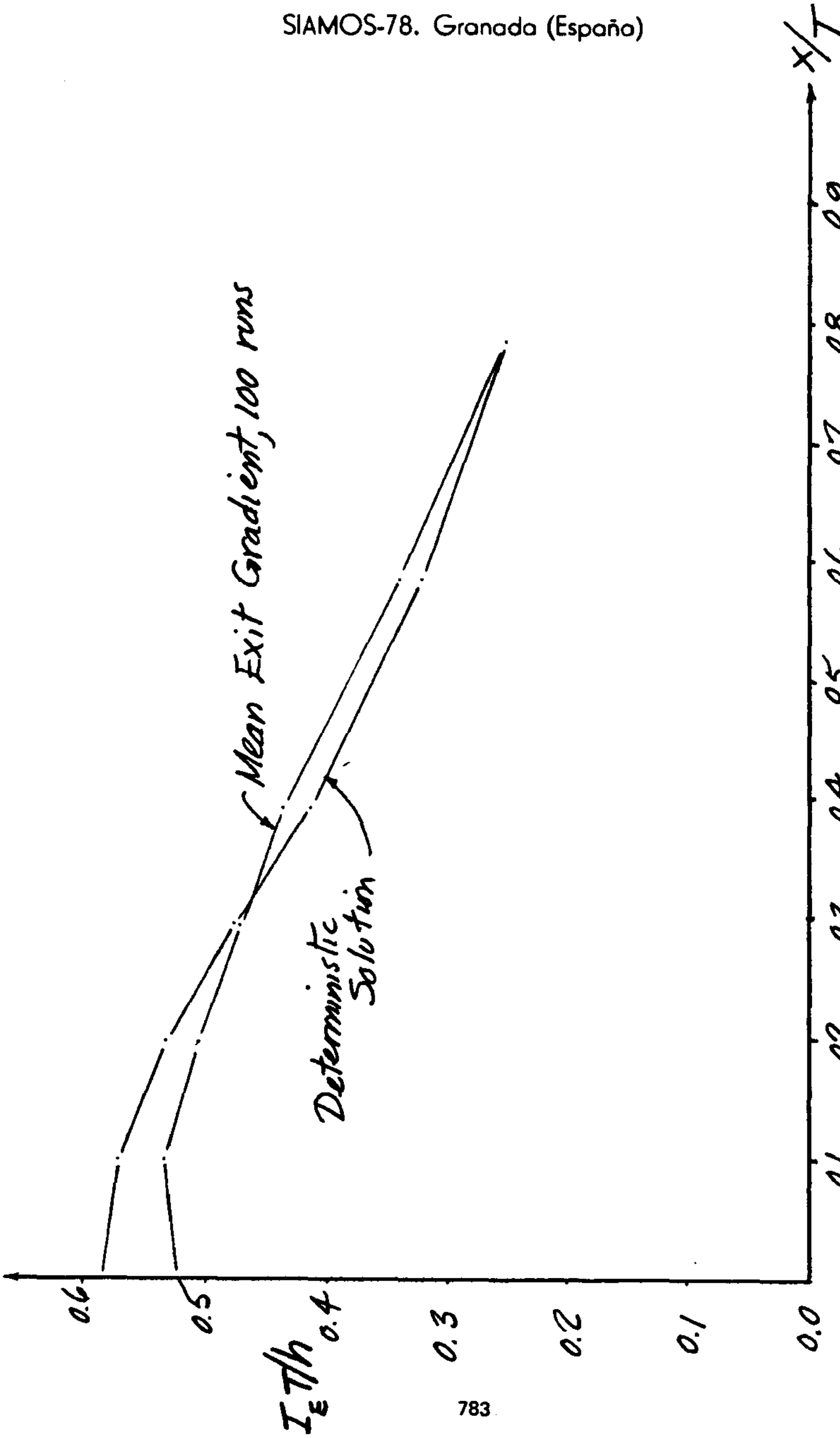


Figure 5: Exit Gradient (100 runs)

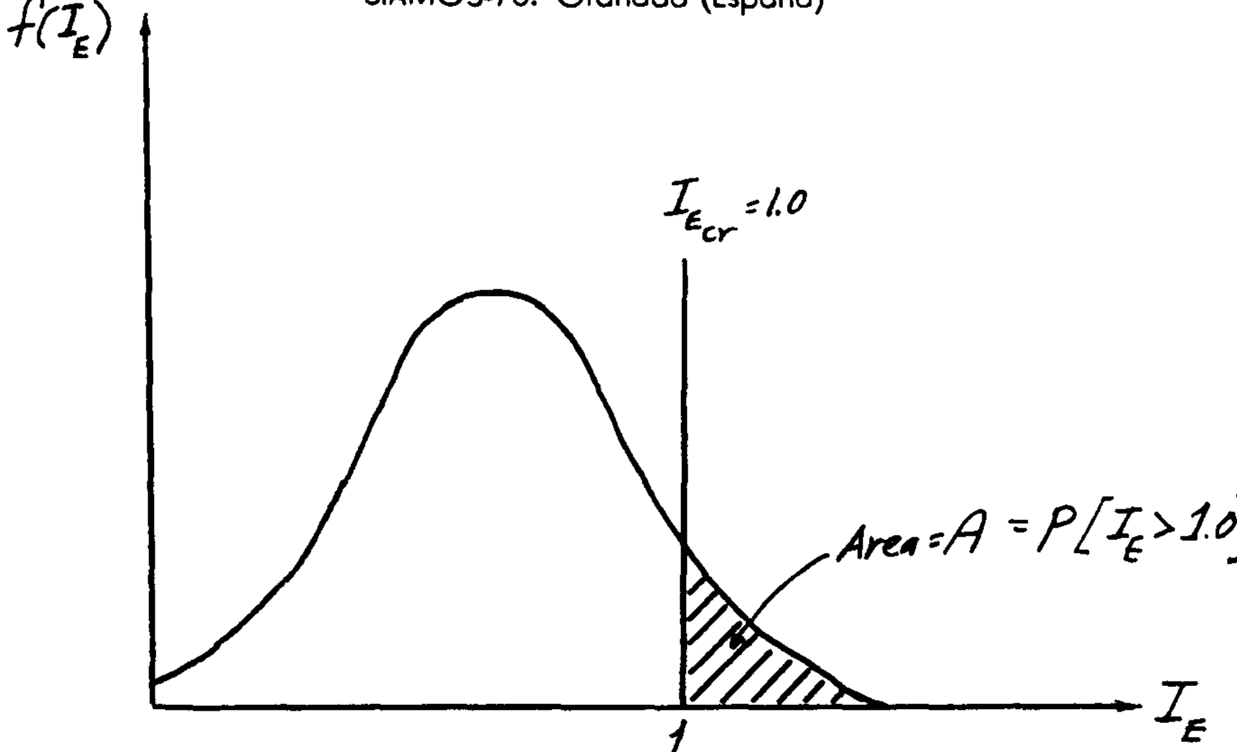


Figure 6: Probability of Piping

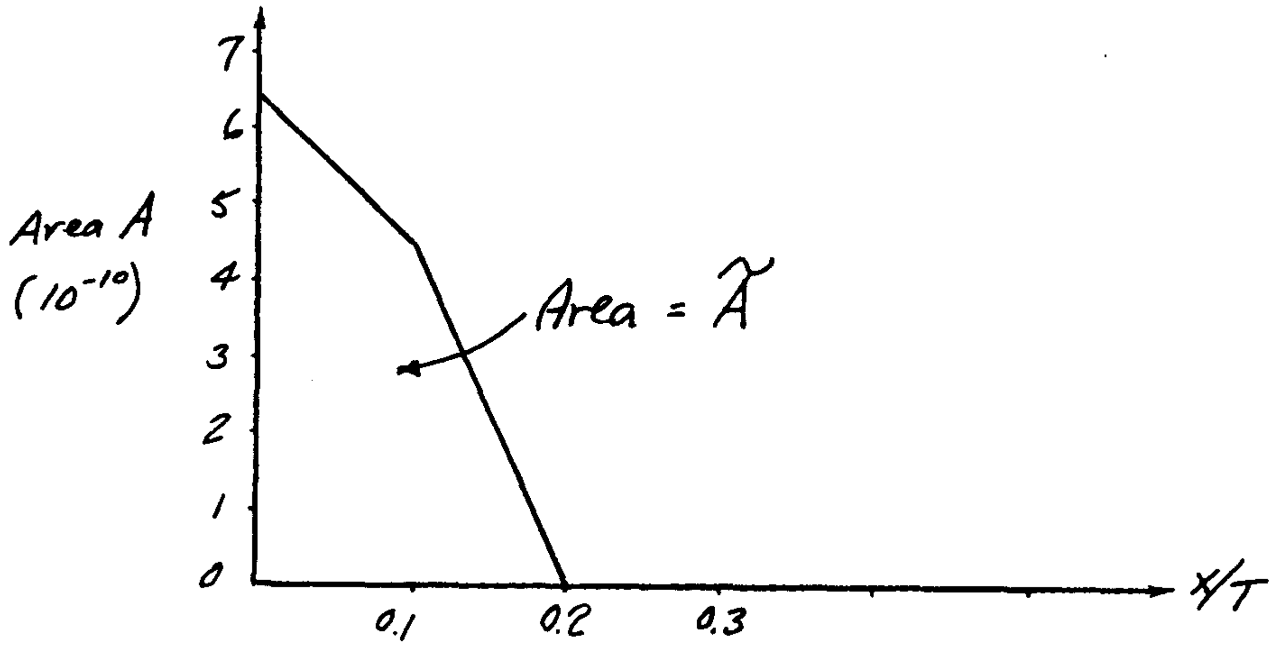


Figure 7: Probability of Piping vs Lateral Distance from Piling

Having a measure of the probability of piping at all points on the downstream discharge surface, the next matter at hand is to estimate the overall probability of the piling system, of length L.

Taking $\bar{d}$ to be the mean diameter of a soil particle in centimeters (such as the classical d_{10} size) the number of particles in one square meter of the discharge surface will be

$$N = (100/\bar{d})^2 \tag{4}$$

Defining $\bar{A}$ as the average probability of failure due to piping over the lateral distance of $x = 0.2T$, as shown to be the region of interest in Figure 7, we have that the probability of piping for the length of sheetpiling is

$$P_p = \bar{A}(0.2T)NL$$

Noting that $0.2\bar{A} = \tilde{A}$ or the area so designated in Figure 7, we have for the overall probability of failure

$$P_p = \tilde{A}TL(100/\bar{d})^2 \tag{5}$$

where T and L are in meters and $\bar{d}$ is in centimeters.

EXAMPLE

Given: $s/T = 0.55$, $h/T = 0.50$, $T = 10m$, $L = 100m$, medium sand with $\bar{d} = 1.0mm$.

Find: probability of piping.

Solution: Obtain the deterministic solution for the exit gradient from Equation (2) for some values of x/T. These are given in the second column of Table 2. The corresponding values for the standard deviation, S_{I_E}, with a coefficient of variation of 45% are given in column 3.

The standard normal variates for the critical gradient $I_E = 1$, $z = (1 - I_E)/S_{I_E}$, are listed in column 4 of the Table.

The corresponding normal variates are tabulated in column 5. These can be had from the expression (Harr, 1977)

$$P[I_E \geq 1] = \frac{1}{z}(2\pi)^{-1/2} \exp[-z^2/2] \tag{6}$$

These are translated into $\bar{A}$ values in column 6 and finally into $\tilde{A}$ in column 7. Hence, for the given conditions, we have from Equation (5)

$$P_p = \tilde{A}TL(100/\bar{d})^2 = 6.63x10^{-11}(10m)(100m)(1x10^6)/m^2 = 6.63x10^{-2} \text{ or } 6.6\%$$

Thus, the reliability of the system is 93.4%. It is noted that the factor of safety for this case is 1/0.27 = 3.7.

In Table 3 are given some factors of safety (F.S.), their corresponding probabilities of piping (P_p) and their reliability ($1 - P_p$).

Khosla, Bose and Taylor (1954) recommend that the following factors of safety be applied as critical values of exit gradients: gravel, 4 to 5; coarse sand, 5 to 6: fine sand, 6 to 7.

SUMMARY AND CONCLUSIONS

a) A hybrid finite-element, probabilistic model was proposed to determine the probability of failure due to piping in a non-homogeneous layer exhibiting a single sheetpile cut-off.

b) It was found that the distribution of the exit gradient at a point on the discharge surface can be represented as a normal distribution with a mean value equal to the deterministic exit gradient and with a coefficient of variation of 45%.

c) The critical zone for piping is along the discharge surface within a distance of 0.2T from the piling, where T is the thickness of the homogeneous layer.

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Khosla, R. B. A. N., N. K. Bose, and E. McK. Taylor. 1954. Design of Wiers on Permeable Foundations. Central Board of Irrigation. New Delhi, India.

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Table 1

Element Number (see Figure 2)	2	1/4	3/6	5/8	7/66	65	67
Mean exit gradient $\bar{x}$	0.533	0.544	0.516	0.480	0.443	0.339	0.332
Standard deviation s	0.232	0.226	0.205	0.163	0.162	0.123	0.097
Coefficient of variation V_c	44%	42%	40%	34%	37%	36%	29%
Coefficient of kurtosis β_2	2.54	3.18	3.75	2.88	2.69	2.54	2.63
Coefficient of skewness β_1	0.45	0.62	0.85	0.06	0.19	0.44	0.45
Deterministic homogeneous value, I_E	0.595	0.580	0.538	0.483	0.422	0.326	0.321

Table 2

1 x/T	2 I_E	3 S_{I_E}	4 z	5 $P[I_E>1]$	6 $\bar{A}_i$	7 $\tilde{A}_i$
0.0	0.27	0.12	6.08	$6.18(10^{-10})$	$4.85(10^{-10})$	$4.85(10^{-11})$
0.1	0.26	0.12	6.17	$3.51(10^{-10})$	$1.78(10^{-10})$	$1.78(10^{-11})$
0.2	0.25	0.11	6.82	$0.05(10^{-10})$	$0.02(10^{-10})$	-
0.3	0.22	0.10	7.80	-		
						$\tilde{\lambda} = 6.63(10^{-11})$

Table 3

Soil	$\bar{d}$	s/T	h/T	I_E	F.S.	T	P_n (% for 100m)	Reliability
Gravel	25mm	0.55	0.50	0.27	3.73	10m	<0.1	>99.9
Coarse Sand	5mm	0.55	0.50	0.27	3.73	10m	0.3	99.7
Fine Sand	1mm	0.55	0.50	0.27	3.73	10m	7.8	92.2
Fine Sand	1mm	0.55	0.50	0.27	3.73	5m	3.9	96.1
Gravel	25mm	0.55	0.55	0.30	2.95	5m	0.5	99.5
Coarse Sand	5mm	0.55	0.55	0.30	2.95	5m	3.2	96.8
Fine Sand	1mm	0.55	0.55	0.30	2.95	5m	≈100	0
Gravel	25mm	0.70	0.65	0.26	3.91	10m	<0.1	>99.9
Gravel	**25mm**	0.70	0.80	0.31	3.18	10m	17.4	82.6
Fine Sand	1mm	0.85	0.90	0.25	3.94	10m	0.4	99.6