AQUIFER PARAMETER IDENTIFICATION BY USING DIGITAL SIMULATION MODELS

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ABSTRACT : A digital model simulating unsteady-state radial flow to partially penetrating wells in unconfined anisotropic aquifers is applied to analyze test-data from an aquifer test carried out in the alluvial plains of Northern India. The parameters identified are the lateral permeability, anisotropy, specific storage and specific yield of the aquifer. Complex aquifer conditions can be simulated by digital models. It requires fewer number of observation wells for a digital model than those required for analytical methods.

RESUME : Un modèle digital qui simule un flux vacillant et radial aux puits qui sont partiellement pénétrés dans les aquifères illimités, est appliqué pour analyser les résultats d'une épreuve aquifère qu'on a faite dans les plaines du nord de l'Inde. Les paramètres qu'on a identifiés, sont la perméabilité latérale, l'anisotropie, l'emmagasinage spécifique et le produit spécifique de l'aquifère. A l'aide de ce modèle, on a pu simuler les conditions aquifères complexes. D'ailleurs, dans ce modèle, on a besoin d'observations moins nombreuses que celles qu'on doit employer dans les méthodes analytiques.

RESUMEN : Se aplica un modelo digital, que simula el flujo radial en estado transitorio con pozos parcialmente penetrantes en acuíferos libres anisótropos, para analizar los resultados de un bombeo de ensayo efectuado en las llanuras aluviales del norte de la India. Los parámetros identificados son : permeabilidad lateral, anisotropía, almacenamiento específico y caudal específico del acuífero. Los modelos digitales pueden simular las condiciones complejas de los acuíferos reales e incluso se requiere para ello menor número de piezómetros de observación que en el caso de interpretaciones analíticas.

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1. INTRODUCTION

In the conventional approach to analyze aquifer test-data we resort to type-curve methods which in turn are based on analytical solutions (WALTON, 1970). However, quite often the conventional methods do not yield consistent results, because existing type-curve model aquifers do not adequately cover and describe all prevailing aquifer conditions. In this context, the need for application of digital computer techniques to aquifer test-data analysis assumes importance (WALTON, 1976). The obvious advantage in this approach, is the facility to consider complex hydrogeologic conditions which have either to be idealized or ignored in the analytical type-curve techniques. The approach here would be to seek numerical solutions of the hydraulic heads in the aquifer in response to a test-well discharging at a constant rate, with assumed values of aquifer parameters, and then comparing those model responses with observed field data. In the present work the application of such techniques is illustrated in the specific context of aquifer test situations commonly encountered in the alluvial plains of Northern India.

The alluvial plains in Northern India are underlain by stratified formations with relatively more permeable strata interbedded between relatively less permeable strata which are however limited in their later extent. Tube-wells pumping from these aquifers are only screened against the relatively more permeable strata and are therefore in effect partially penetrating wells. While carrying out aquifer tests for identifying the parameters characterizing these aquifers, the test-well is usually run at a constant discharge and time-drawdown data at one or two observation wells screened in test-well screen interval are recorded. The time-drawdown data do resemble those under leaky artesian aquifer conditions. However piezometric tubes tapping water table depths show that there is a lowering of the water table. Due to this fact and along with the fact that the aquifer formations are stratified, it may be more appropriate to treat the aquifer system as a single anisotropic aquifer under water table conditions, and the well to be partially penetrating. The aquifer anisotropy will take care of the stratifications.

Analytical methods to analyze data of tests carried out in aquifer conditions similar to the one encountered in the alluvial plains of Northern India have been reported elsewhere (BENNET, REHMAN, SHEIK, AND ALI SABIR, 1967). These methods however ignore the compressibility of the aquifer and assume that the well discharge is derived entirely from water table storage. Also, to apply these methods time-drawdown data at a number of pairs of observation wells spaced over the full radius of influence of test-well are required. Besides the above two limitations, the lateral permeability that is evaluated is also subject to errors since the flow to well is not strictly convergent in the depth interval and radial distances at which the drawdown are recorded (LAKSHMINARAYANA AND RAJAGOPALAN,1977).

In the present study numerical solutions are sought for the unsteady-state radial flow to a partially penetrating well pumping at a constant rate from an unconfined anisotropic aquifer. The solution technique is the iterative alternate direction implicit method (PEACEMAN AND RACHFORD, 1955). The computing algorithm is a modified version of the one that has been earlier reported (PRICKETT AND LONNQUIST, 1971). The numerical solution presented here is an extension of an earlier work where the aquifer was treated to be isotropic and incompressible and the well to be fully penetrating (STRELTSOVA AND RUSHTON, 1973). The digital simulation model is applied to analyze data from an aquifer test carried out by the Water Resources Directorate, Punjab, India. The test-data comprises of time-drawdown data at two pairs of observation wells, one of the pair tapping well-screen depths and the other the water table depths. The aquifer parameters that are identified are the lateral permeability, anisotropy, specific storage and specific yield of the aquifer. These parameters were repeatedly adjusted in the model till the model response matched sufficiently well with the field data. An added advantage of such a digital simulation approach is that one quite often can get away with a fewer number of observation wells than are required for analytical methods and thereby, economizing on the aquifer test expenditure. The application of digital simulation technique in analysing aquifer test-data that has been reported here is an extension of an earlier work where the test-well was fully penetrating and the aquifer was treated to be isotropic and incompressible (RUSHTON AND BOOTH, 1976).

2. THEORY

In unconfined aquifers water is released by compaction of aquifer, expansion of water and gravity drainage at free surface (WALTON, 1960). Water release by gravity drainage at free surface has been characterised as delayed yield (BOULTON, 1954). Boulton gave the theory of unsteady flow to fully penetrating wells in unconfined isotropic aquifers (BOULTON, 1963). Based on this theory type-curve solution methods to analyze test-data have been reported (PRICKETT, 1965). Boulton's delayed yield theory is identified with the influence of flow in the unsaturated portion or in other words in the dewatered zone. However, later it was demonstrated that the unsaturated flow plays a subordinate role in the delayed yield process, and a conclusion was arrived at that it is the free surface and flux which have an exponential change with time, and that specific yield remains a constant (STRELTSOVA, 1973). In line with this conclusion, analytical solution for unsteady flow to partially penetrating wells in unconfined anisotropic aquifers have been reported where, the aquifer is treated as a compressible system, the phreatic surface is treated as a moving material boundary, the specific storage and specific yield are treated constants, and the well is assumed to be of infinitisimal diameter (Neuman, 1974).

A schematic representation of a partially penetrating well pumping at a constant rate from an unconfined aquifer of infinite lateral extent, exhibiting simple two-dimensional anisotropy and resting over an impermeable horizontal bed is shown in Figure 1. If it is assumed that a) the flow exhibits radial symmetry, b) water is released by compaction of aquifer, expansion of water and gravity drainage at water table, c) radial hydraulic gradient is constant over the full length of well screen, d) water table drawdown is a small fraction of the saturated thickness of aquifer, and e) prior to pumping, the potential throughout the aquifer is same, the partial differential equation governing the unsteady-state flow and the boundary conditions may be written as,

$$P_r \left[\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right] + P_z \left[\frac{\partial^2 h}{\partial z^2}\right] = S_s \frac{\partial h}{\partial t} \quad (1)$$

$$h(\gamma, z, 0) = h_0 \quad (2)$$

$$h(\infty, z, t) = h_0 \quad (3)$$

$$\frac{\partial h}{\partial z}(\gamma, m, t) = -S_y \frac{\partial h}{\partial t}(\gamma, m, t) \quad (4)$$

$$\frac{\partial h}{\partial z}(\gamma, 0, t) = 0 \quad (5)$$

$$\frac{\partial h}{\partial}(\gamma_w, z, t) = 0 \text{ for } 0 < z < z_1 \text{ and } z_2 < z < m \quad (6a)$$

$$= Q/(2\pi\, \gamma_w P_\gamma\, l) \text{ for } z_1 \leq z \leq z_2 \quad (6b)$$

where, P_r and P_z are lateral and vertical permeabilities; S_s and S_y are specific storage and specific yield; m is saturated thickness; h is hydraulic head; h_0 is initial hydraulic head; r is radial coordinate; z is vertical coordinate; t is time coordinate; r_w is radius of well; l is screen length; and z_1 and z_2 are bottom and top elevations of well screen.

A numerical solution is sought for the Equation (1) under the above specified boundary conditions.

3. DIGITAL SIMULATION MODEL

A two-dimensional finite difference grid with the columns and rows co-axial with the z and r axes respectively is designed to model the aquifer. The radius represented by each node along rows is a constant multiple α, of that represented by the preceeding node. If radial distances are taken in the logarithmic scale, the nodes along rows occur at equally spaced intervals $\log_e \alpha$. The inner most column represents a radial distance r_w. Δz, the distance between nodes along columns was so chosen as to incorporate in the model identical well penetration as existing in the field test-well. To simulate effectively the infinite lateral extent of the aquifer, the number of columns in the model were made to be sufficiently large so that the potential heads in the aquifer were not affected by the presence of the boundary edge. In the finite difference grid, each branch is assigned a hydraulic conductance value, and each node is assigned a storage factor value. Hydraulic conductance to be assigned to branches along the rows is computed as the product of lateral permeability of the aquifer and the area of cross-section normal to the radial direction divided by the length of flow parallel to the radial direction. In the case of branches along columns, the hydraulic conductance is computed as the product of vertical permeability of the aquifer and the area of cross-section normal to vertical flow divided by the length of flow parallel to the vertical direction. The storage factor to be assigned to nodes in the internal rows and the bottom most row is computed as the product of the specific storage of the aquifer and the volume of aquifer segment represented by each of them. In the case of the top most row which represents the water table, storage factor to be assigned to nodes in it is computed as the product of the specific yield of the aquifer and the area of cross-section to vertical flow represented by each of those nodes.

Either from a formal mathematical treatment, substituting finite difference approximations for the derivatives, or from a physical standpoint involving Darcy's law and the principle of conservation of mass, the finite difference equation for each of the nodes can be derived in terms of the hydraulic head at the node under consideration, the hydraulic heads at the adjacent nodes the hydraulic conductance assigned to branches connecting the node under consideration and the adjacent nodes, and the storage factor assigned to the node under consideration. These equations are written in the implicit form where the space derivatives are replaced by their finite difference equivalent at the time level at which potentials are to be calculated and the time derivative is replaced by a backward difference approximation. This implicit approximation has been proved to be unconditionally stable regardless of the time increment (SMITH, 1965). At the nodes representing the well screen the well discharge is simulated, with discharge at each of them being proportional to the length of the well screen represented by them.

The finite difference equation written for all the nodes in the finite difference grid gives rise to a system of simultaneous equations which has to be solved for the hydraulic heads. The iterative alternate direction implicit method (PEACEMAN AND RACHFORD, 1955) was adopted to solve this system of simultaneous equations. An available computing algorithm for this method (PRICKETT AND LONNQUIST) was suitably modified and used. The computer program written in Fortran IV language was run on IBM 7044/1401 system.

4. APPLICATION OF THE MODEL

The digital model was applied to analyze data from an aquifer test carried out by the Water Resources Directorate, Punjab. The test well was a gravel pack well of radius 0.102 metres, and was screened against the relatively more permeable strata identified by electric logging of the test-well bore. Two pairs of observation wells, O_1; Pz_1 and O_2; Pz_2, at radial distances 18.44 metres and 61.26 metres respectively, were used to record the time-drawdown data during the test-period. O_1 and O_2 tapped well screen depths and Pz_1 and Pz_2 tapped water table depths. The test-well was run at a constant discharge of 0.032 metre^3 per second and duration of test was about 3000 minutes. The time-drawdown data recorded at the observation wells are shown in Table 1 and a graphical plot can be seen from Figure 2.

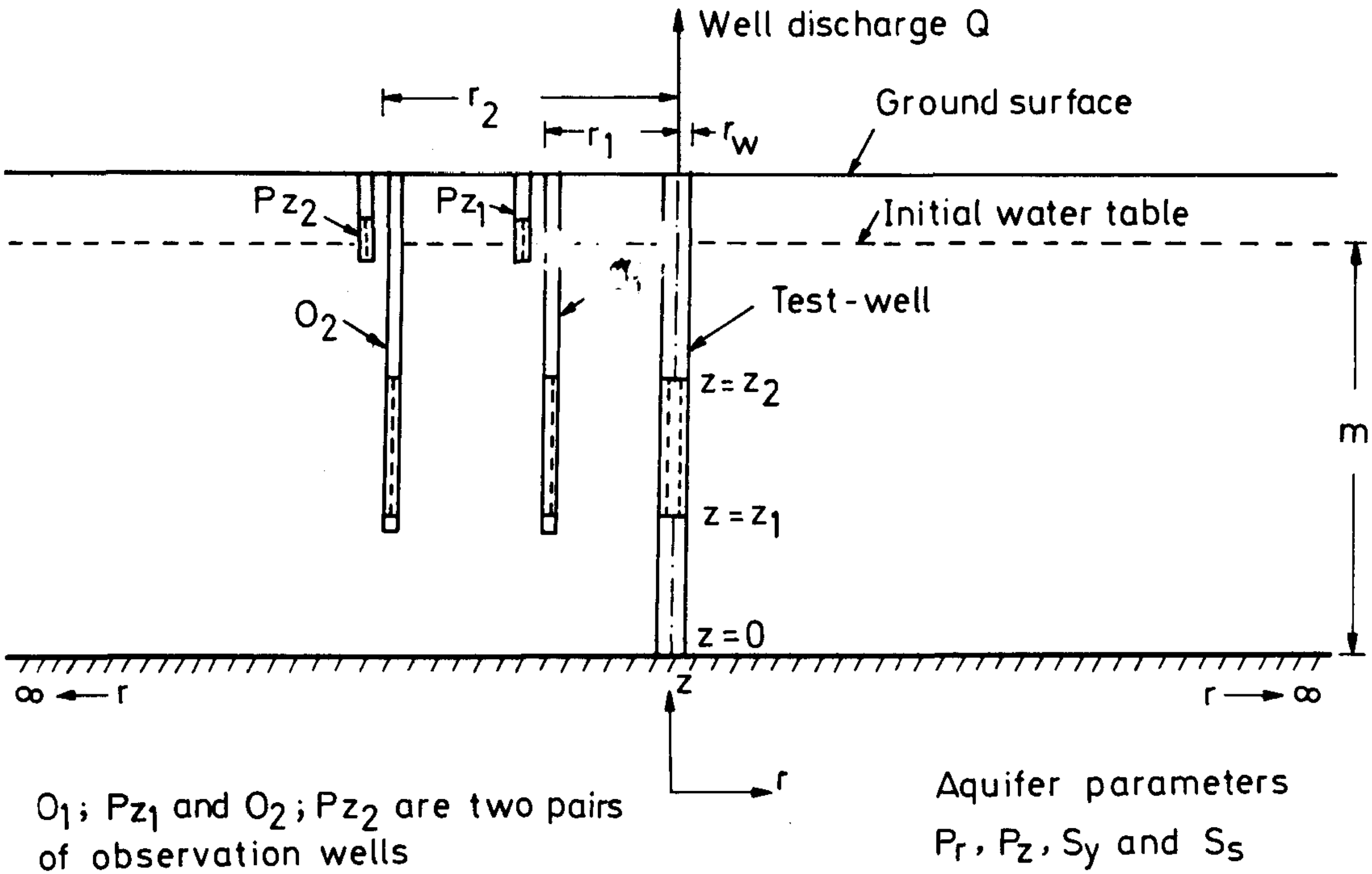
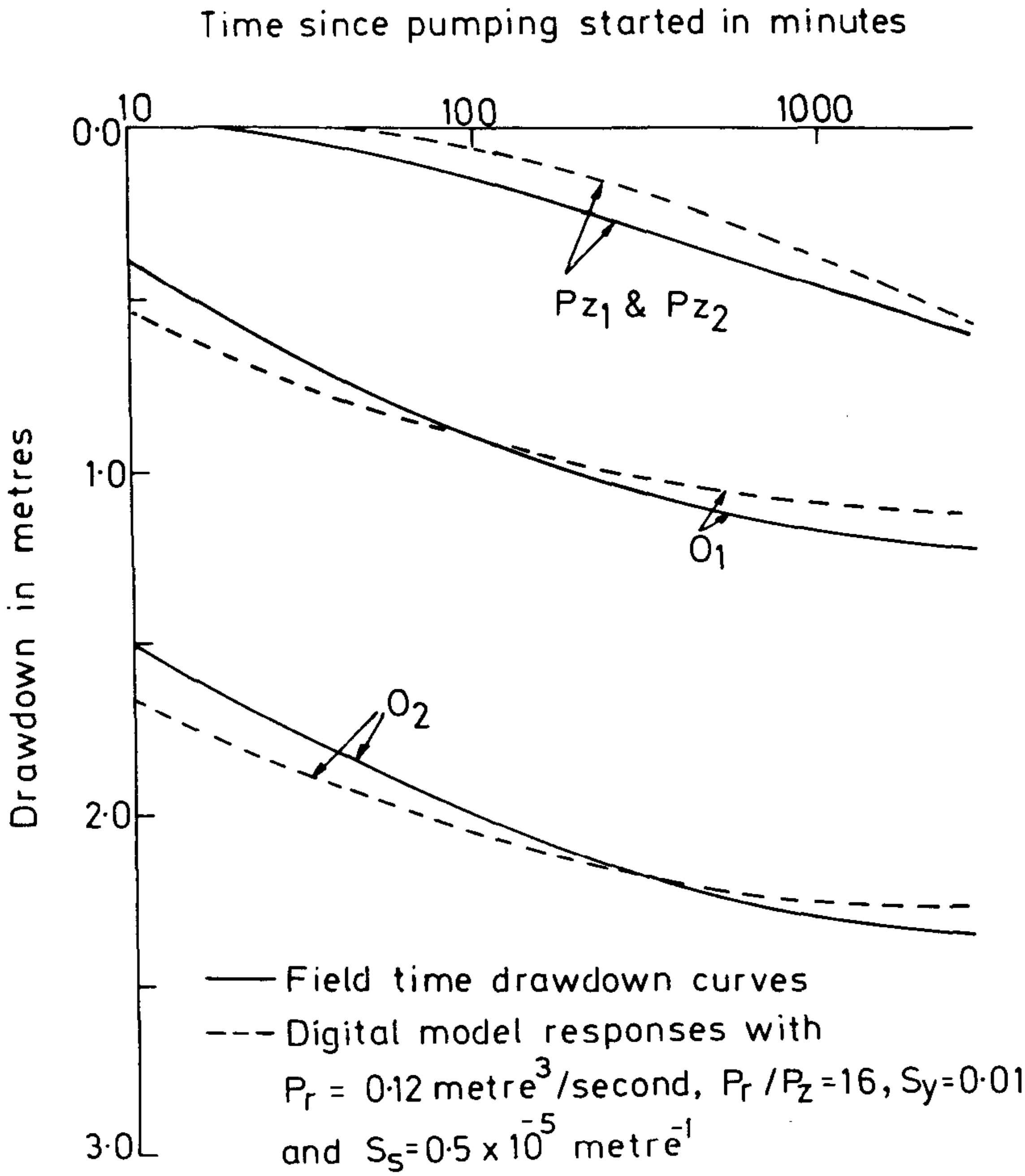


FIG.1 SCHEMATIC REPRESENTATION OF AQUIFER AND WELL CONDITIONS FOR THE UNSTEADY-STATE FLOW MODEL



O_1 and O_2 tap well-screen depths

Pz_1 and Pz_2 tap water table depths

FIG.2 VARIATION OF DRAWDOWN WITH TIME

The drawdown data at well-screen depths do resemble that to be found in leaky artesian aquifers, and this suggests the possibility that the aquifer test-data may be analyzed by type curve methods available for the case of leaky artesian aquifers. Such an analysis would imply that the aquifer at test-well screen depths is under leaky artesian isotropic aquifer conditions and is overlain above the top of well-screen by a source bed at a constant potential. Observation wells Pz_1 and Pz_2 used in the field aquifer test, however show that there is a lowering of water table during pumpage. This aspect of the water table behaviour would have been missed if these two observation wells were not installed. The electric logging of the test-well bore also indicates that the aquifer is a stratified formation with relatively less permeable strata. The pattern of drawdown at well screen depths is probably explained by the fact that the test-well is screened at sufficient depths below the water table. Under these conditions it should be more appropriate to treat the aquifer test-situation as one where a partially penetrating well is pumping at a constant rate from a single anisotropic aquifer under water table conditions. The effect of stratifications will be taken care of by the anisotropy.

To model the aquifer the saturated thickness of aquifer was taken as 150 metres. A variable size z, was so chosen as to incorporate identical well penetration as existing in the field test-well. The radius represented by each node along rows was adopted as $\sqrt{2}$ times that of the preceeding node. The finite difference grid had 12 rows and 31 columns. At the nodes representing the well-screen in the innermost row of the model, the well discharge was distributed in proportion to the fractional screen length represented by each of them. The initial time increment chosen was 1 minute and for successive time steps, the time increment was increased by a factor of 1.2. Total number of time steps adopted was 35.

Repeated trial runs were made with various combinations of values for lateral permeability, anisotropy, specific storage and specific yield. Model response of time-distribution of drawdown at well-screen depths and water table drawdown were obtained for each trial run at the two radial distances at which the pairs of observation wells were located in the field aquifer test. The model response for each of these trial runs were compared with field drawdown data and the particular combination of P_r, P_r/P_z, S_s and S_y in the model for which this comparison gives the best possible match was identified. It was found that adopting lateral permeability P_r, as 0.12 x 1 $^{-3}$, anisotropy P_r/P_z, as 16, specific storage S_s, as 0.5 x 10^{-5}, and specific yield S_y, as 0.01 in the model, the model response agreed reasonably well with the field drawdown data at both the pairs of observation wells. The model response for this case is also reported in Figure 2.

5. CONCLUSION

The aquifer test-situations commonly encountered in the alluvial plains of northern India are partially penetrating wells pumping from unconfined anisotropic aquifers. Conventional type-curve methods of analysis of the test-data are often not appropriate. On the other hand the test-data may be analysed using a digital simulation model where the model parameters can be repeatedly adjusted till the model response matches sufficiently well with the field data. In the digital model a numerical solution is sought for the time distribution of drawdown at well screen depths and water table drawdown in response to a partially penetrating well pumping at a constant rate from unconfined anisotropic aquifer of infinite lateral extend and resting over an impermeable bed. The aquifer parameters that are identified are the lateral permeability, anisotropy, specific storage and specific yield. The field test-data requirements are time-drawdown data at two or three pairs of observation wells, one of the pair tapping well-screen depths and the other tapping water table depths. The digital simulation model has been applied to analyze data from an aquifer test carried out by the Water Resources Directorate, Punjab, India. An added advantage of such digital simulation approach over analytical methods is that a fewer number of pairs of observation wells are required thereby leading to considerable saving in aquifer test expenditure.

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TABLE 1. TIME-DRAWDOWN DATA

Time after pumping started t in minutes	Drawdown in Feet			
	O_1	Pz_1	O_2	Pz_2
2	3.59		0.49	
4	3.76		0.61	
6	3.93		0.90	
8	4.07		1.12	
10	4.34		1.28	
20	5.09		1.82	
30	5.42		2.13	
60	6.01	0.47	2.65	0.46
120	6.71		3.07	
180	6.90	0.83	3.28	0.81
300	7.04	0.95	3.44	0.94
420	7.22	1.10		1.06
540	7.27	1.18	3.64	1.14
720	7.29	1.30	3.71	1.26
900	7.30	1.35	3.74	1.33
1260	7.36	1.50	3.78	1.44
1500	7.55	1.58	3.88	1.54
1620	7.65	1.69	3.99	1.60
1860	7.80	1.77	4.07	1.69
2100	7.82	1.87	4.12	1.79
2340	7.84	1.92	4.20	1.83
3060	7.86	1.96	4.20	1.92